DOCUMENT RESUME

ED 325 332 SE 051 668

AUTHOR Natour, Denise M.; And Others
TITLE Instructor's Guide to the CCA Basic Skills Mathematics Curriculum.
INSTITUTION Illinois Univ., Urbana. Computer-Based Education Research Lab.
PUB DATE Apr 90
NOTE 239p.
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS Adult Education; *Basic Skills; Computer Assisted Instruction; *Computer Uses in Education; Curriculum Guides; *Elementary School Mathematics; Learning Activities; Mathematics Education; Pretests Posttests; *Remedial Instruction; Resource Materials; *Secondary School Mathematics

ABSTRACT

The curriculum consists of 16 units covering: (1) addition, subtraction, multiplication, and division of whole numbers; (2) common fractions; (3) mixed numbers; and (4) decimals. Each unit consists of a pretest, assignment of lessons as needed, and a posttest. Paper-and-pencil worksheets that accompany the computer lessons are an integral part of the curriculum. Each unit includes lesson name, computer file where the lesson can be found, author(s) and affiliation, objectives, description, interaction, intended audience, and completion time. This curriculum is designed for, and delivered by, the SYS 4 instructional management system. When a student does not earn PREtest credit, SYS 4 can either make an automatic assignment of lessons or alert the instructor to make an assignment. When a student does not earn credit on a POSTtest, intervention is always required. Instructors can choose to have a student re-do lessons, do other lessons, work with paper-and-pencil materials, or work with a tutor. Sample computer generated worksheets and an alphabetical list of files used in this curriculum are appended. (KR)

**********
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
**********

ED325332

# Instructor's Guide

## CCA Basic Skills

# MATHEMATICS

## Curriculum

"PERMISSION TO REPRODUCE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY

J. R. Dennis

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

V This document has been reproduced as received from the person or organization originating it.

Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

**Courseware & Curriculum Applications**
**Computer-based Education Research Laboratory**
**University of Illinois at Urbana-Champaign**

**April 1990**

SE051668

# INSTRUCTOR'S GUIDE TO THE CCA BASIC SKILLS MATHEMATICS CURRICULUM

April 1990

For further information, contact:

**Martin A. Siegel**
Assistant Director, CERL

Documentation by:

**Denise M. Natour**
**Amy S. Baker**
**Dorothy Z. Silver**

University of Illinois
Computer-based Education Research Laboratory
103 South Mathews Avenue
Urbana, Illinois 61801

Copyright (c) April 1990
by The Board of Trustees of the University of Illinois
at Urbana-Champaign

All rights reserved.

No part of this book may be reproduced in any form or by any means
without permission in writing from the authors.

The PLATO® and NovaNET® systems are developments of the University of Illinois.

PLATO® is a trademark of Control Data Corporation (CDC).

NovaNET® is a service mark of University Communications, Inc. (UCI).

## TABLE OF CONTENTS


PREFACE ..... vi-xiv

**Unit 1: Basic Addition** ..... 1

Adding Lengths ..... 2

Facts Practice [Addition] ..... 4

**Unit 2: Advanced Addition** ..... 6

Facts Practice [Addition] ..... 7

Adding and Carrying ..... 8

Checkup on Addition ..... 10

Speedway! [Add and Subtract] ..... 12

**Unit 3: Subtracting without Borrowing** ..... 15

Facts Practice [Subtraction] ..... 16

Speedway! [Add and Subtract] ..... 18

**Unit 4: Subtracting with Borrowing** ..... 19

Subtraction with Sticks ..... 20

Facts Practice [Subtraction] ..... 22

Torpedo [Whole Numbers] ..... 23

**Unit 5: Learning to Multiply** ..... 25

Facts Practice [Multiplication] ..... 26

Rubber Stamp ..... 28

Arrays ..... 30

Multiplying by a One-Digit Number ..... 32

Multiplication Is ... ..... 34

Speedway! [Add, Subtract, Multiply, Divide] ..... 36

**Unit 6: Multiplying by Two or More Numbers** . . . . . 39

Facts Practice [Multiplication] . . . . . 40

More Arrays . . . . . 41

Ant War! . . . . . 43

Multiplying by Numbers Ending in Zero . . . . . 45

Multiplying by Two-Digit Numbers . . . . . 47

Speedway! [Add, Subtract, Multiply, Divide] . . . . . 49

**Unit 7: Dividing by Ones Numbers** . . . . . 50

Facts Practice [Division] . . . . . 51

Division Is ... . . . . 53

Gimme! . . . . . 55

Dividing by Trying Numbers to See If They Fit . . . . . 57

Introduction to Long Division: Part 1 . . . . . 59

Speedway! [Add, Subtract, Multiply, Divide] . . . . . 61

**Unit 8: Dividing by Tens or Hundreds Numbers** . . . . . 62

Facts Practice [Division] . . . . . 63

Introduction to Long Division: Part 2 . . . . . 64

Introduction to Long Division: Part 3 . . . . . 66

Speedway! [Add, Subtract, Multiply, Divide] . . . . . 68

**Unit 9: Consolidation #1** . . . . . 69

Pinball Number Facts Drill . . . . . 70

**Unit 10: What Is a Fraction?** . . . . . 72

Cut and Paint . . . . . 74

Fraction Notation . . . . . 76

Lights . . . . . 78

Lights: Equivalent Fractions 1 . . . . . 80
Checkup: Cut and Paint . . . . . 82
Checkup: Lights . . . . . 84
Pizza: Mixed Numbers Introduction . . . . . 86
Try These: Fractions and Mixed Numbers . . . . . 88
Try These: Equivalent Mixed Numbers . . . . . 90
Vocabulary: Whole Number, Fraction, Mixed Number . . . . . 92
Sort Numbers by Form . . . . . 94
Checkup: Mixed Numbers with Pizza . . . . . 96
Checkup: Equivalent Mixed Numbers . . . . . 98
Lights: Equivalent Fractions 2 . . . . . 100
Boxes: Name Equivalent Fractions . . . . . 102
Sort Equivalent Fractions . . . . . 104
Sort Equivalent Mixed Numbers . . . . . 106
Try These: Equivalent Fractions . . . . . 108
Introduction to Simplest Terms . . . . . 110
Pizza: Common Denominator . . . . . 112
Checkup: Sort Equivalent Fractions . . . . . 114
Checkup: Try These Equivalent Fractions . . . . . 116
Checkup: Simplest Terms . . . . . 118
**Unit 11: Adding and Subtracting Fractions with Like Denominators** . . . . . 120
Mixed Number Notation . . . . . 121
Mixed Number Answer Format . . . . . 123
Add and Subtract Mixed Numbers . . . . . 125
Simplifying Answers to Addition Problems . . . . . 127
Standard Form Practice . . . . . 129

Addition Practice, Simplifying Answers . . . . . 131
Subtraction with Borrowing . . . . . 133
Addition and Subtraction Drill [Like Denominators] . . . . . 135
Addition and Subtraction with Simplifying and Borrowing . . . . . 137
Checkup: Standard Form . . . . . 139
Checkup: Add and Subtract Mixed Numbers . . . . . 141
**Unit 12: Adding and Subtracting Fractions with Unlike Denominators** . . . . . 143
Paint Addition: The Computer Paints . . . . . 144
Addition by Using an Easier Problem 1 . . . . . 146
Addition by Using an Easier Problem 2 . . . . . 148
Paint Addition: You Paint . . . . . 150
High Wire . . . . . 152
Addition and Subtraction Drill [Unlike Denominators] . . . . . 154
Checkup: Paint Addition . . . . . 156
Checkup: Unlike Denominators . . . . . 158
Equivalent Fractions Generalized . . . . . 160
Addition with Equivalence Sets . . . . . 162
Add and Subtract with Equivalence Sets . . . . . 164
Strategies for Listing Equivalence Sets . . . . . 166
**Unit 13: Multiplying and Dividing Fractions** . . . . . 168
Introduction to Multiplying Mixed Numbers . . . . . 169
Multiplication Practice with Pictures for Help . . . . . 171
Multiplication of Fractions . . . . . 173
Strategies for Multiplying Mixed Numbers . . . . . 175
Mixed Number Multiplication Practice . . . . . 177
Practice with Fractions . . . . . 179

**Unit 14: Adding and Subtracting Decimals** . . . . . 181

Decimal Darts . . . . . 182

Decimal Skills I: Reading and Writing Decimals . . . . . 184

Decimal Skills IIa: Adding Decimals . . . . . 186

Decimal Skills IIb: Subtracting Decimals . . . . . 188

Decimal Pinball . . . . . 190

Decimal and Fraction Conversions . . . . . 192

Sort Number Names with Decimals . . . . . 194

Decimal Skills V: Comparing Decimals and Fractions . . . . . 196

**Unit 15: Multiplying and Dividing Decimals** . . . . . 198

Decimal Skills IIIa: Multiplying Decimals . . . . . 199

Decimal Skills IIIb: Dividing Decimals . . . . . 201

Decimal Skills IV: Rounding Decimals . . . . . 203

Math Review Drills . . . . . 205

**Unit 16: Consolidation #2** . . . . . 207

Simplest Terms: Equivalent Fractions . . . . . 209

APPENDIX A: Sample of Off-line Materials

APPENDIX B: List of Files

# Preface

## Introduction

This documentation of the *CCA Basic Skills Mathematics Curriculum* was prepared by staff members of the Courseware & Curriculum Applications (CCA) Group, a unit of the Computer-based Education Research Laboratory (CERL), University of Illinois at Urbana-Champaign.

The curriculum consists of 16 units covering addition, subtraction, multiplication, and division of whole numbers, common fractions, mixed numbers, and decimals. Each unit consists of a pretest, assignment of lessons as needed, and a posttest. Paper-and-pencil worksheets that accompany the computer lessons are an integral part of the curriculum.

## Overview of the SYS 4 Management System

### General Structure

This curriculum is designed for, and delivered by, the SYS 4 instructional management system. When a student does not earn PREtest credit, SYS 4 can either

a. make an automatic assignment of lessons, or
b. alert the instructor to make an assignment.

The decision of which option to use is made when the group is set up, and is a group-by-group decision made by the site manager. In either case, instructors always have options to override the management system for individual students with special needs. Also, whichever option is being used, instructors must see to it that students receive paper-and-pencil practice materials appropriate to the unit (see Curriculum Worksheets below).

When a student does not earn credit on a POSTtest, instructor intervention is always required. It is important at this point that instructors review the test, interview the student, and make a judgment about the next step for the student to take. Instructors can choose to have a student re-do lessons, do other lessons, work with paper-and-pencil materials, or work with a tutor. Instructors may then reassign the posttest (usual), or move the student forward or backward one unit without further testing.

### Assignment Procedures

When students need attention, SYS 4 alerts instructors in one of two ways: either the student signals the instructor when a special message appears on the student's screen (see Figure 1), or, if the student does not signal the instructor, the instructor will see a solid flag next to the student's name on the roster page (see Figure 2).

Instructors can then look at the student's assignment page, viewing information specific to the curriculum (see Figure 3).

Instructors have options to view individual test results. They can see the exact items the student saw and the answers the student gave (see Figure 4). If instructors see a pattern in the types of items missed, they can give an appropriate assignment.

Instructors can also view the record of previous work done to determine any patterns that may be pertinent (see Figure 5).

And finally, instructors can view the standard assignment recommended for the unit. Sometimes only a portion of the lessons available is marked *DO*; the others are indicated by *SKIP*. Instructors may alter the *DO/SKIP* settings according to their judgment of student needs (see Figure 6). Most instructors find the standard assignment appropriate for most students.

Note: For complete instructions on how the SYS 4 management system works, consult the *Instructor's Guide to the SYS 4 Management System* by Dorothy Silver (1988).

## Overview of the Curriculum

### Pretests

Pretests contain eight multiple-choice items (except Consolidation Units 9 and 12, which have 20 items each; see Consolidation Units below). To receive credit for the unit, students must achieve a score of 80% or better.

### Assignment of Lessons

Between 2 and 23 lessons are available in each unit. A core of lessons is always assigned (except for Units 9 and 16, the consolidation units, in which all assignments are left to the instructor's discretion). For many units, however, additional lessons are available for instructors to assign, if they feel the lessons may help a particular student. It is recommended that instructors review each student's test results to determine whether these additional lessons would be helpful.

Note: Lessons assigned only if the instructor chooses to do so are indicated in each unit overview by {curly brackets}. In addition, a special note is made in each lesson description.

### What the Student Sees

When students begin working on lessons, they see only the first lesson in the sequence for that unit. Once they have completed the first lesson, an asterisk appears next to it on their list of lessons, and the second lesson is added to the list. When the second lesson is completed, it, too, is starred and the third lesson appears, and so forth, down the list of lessons. Each starred lesson is available for review if students choose to do so (see Figure 7).

Once all lessons in a unit are completed, students are sent to the unit posttest.

### Curriculum Worksheets

A series of paper-and-pencil worksheets is available, designed to be used in tandem with the computer lessons. These worksheets serve as extra practice for concepts learned in the lessons. They also cover some areas that are not covered in the lessons (particularly in the area of addition and subtraction of whole numbers). Also, they help students transfer skills learned in the lessons to the more standard paper-and-pencil environment. It is intended that students spend approximately half of their time doing computer lessons and half of their time doing paper-and-pencil work.

There are always three sheets of problems for each objective (1A, 1B, 1C). If students do one of the sheets with a sufficiently high score, they do not have to do the other two sheets, but move on to the next level (3A, 3B, 3C). If, however, they have difficulty, then they can practice the same skills using one or both of the other worksheets available in the set.

The sets of worksheets available are listed in each unit overview. Samples of these worksheets can be found in *Appendix A*. Contact your NovaNET distributor for a complete set of worksheets.

Many instructors will have favorite workbooks of their own to use with students, and this is also encouraged. Instructor-selected workbooks may be used in addition to, or even instead of, the SYS 4 Curriculum Worksheets. However, in all cases students should do an adequate amount of paper-and-pencil work in addition to the computer lessons. Instructors should apply standards and methods like those described just above.

### Assignment Sheets

Assignment sheets are offered to help keep track of individual progress. These assignment sheets list the lessons available in each unit, and have a checklist for keeping track of lessons completed and paper-and-pencil work assigned. Samples of these assignment pages are in *Appendix A*. Contact your NovaNET distributor for a complete set of assignment sheets.

**Options at Failure of Posttest**

There are 20 items in each posttest. To earn credit for the unit, students must achieve a score of 80% or better.

When students fail a posttest, they are stopped and their instructor is automatically notified. In all but one unit, the default assignment will be the same as the assignment after failure of pretest. At this point, the instructor is especially urged to review the missed items in the test, talk with the student, examine completed worksheets, and give assignments appropriate for the individual, whether in computer lessons or in worksheets or workbooks.

Unit 12 is the one unit that offers a different assignment of lessons upon failure of posttest. See the Unit 12 overview (page 143) for a description of the lessons assigned after failure of pretest and those assigned after failure of posttest.

## Overview of Lesson Structure

The lessons in this curriculum were selected from much larger curricula developed by the CERL Mathematics Group at the University of Illinois at Urbana-Champaign, by staff at the City Colleges of Chicago, and others. The lessons have been selected and ordered to match the CCA curriculum sequence.

**How Long Is a Lesson?**

Most of these lessons require mastery for completion. When students make mistakes, feedback is given guiding the students to the correct answer. Graphics help show students concrete images of what the numbers are describing. Many of the lessons have several *levels* built in to them, and students are moved up or down levels based on their answers. When the student reaches a high enough level, he obtains credit for the lesson.

Actual completion times for a mastery lesson can vary widely; it depends completely on how much work a student needs. Such lessons are designated *(self-paced)* after the *Completion Time* section in the lesson description to indicate that actual duration will depend on the amount of practice needed.

Some lessons are designated *(open-ended)*, indicating that there is no point at which the lesson automatically ends. These lessons are usually game formats in which students can continue working and practicing until they feel comfortable with the task and ready to move on. To obtain credit, students must work for the required minimum amount of time mentioned in the lesson description.

**Repeated Lessons**

Sometimes the same lesson is used in more than one unit. This is particularly true of the number facts drills. For instance, *Facts Practice [Addition]* is first found in Unit 1, but is also used in Unit 2. Once a lesson has been described, the next time it appears, readers are referred back to the original lesson description.

**Associated Lessons**

Lessons that closely reinforce each other, or that are part of a series of lessons having the same structure and approach, are cross-referenced under the *Associated Lessons* section in each lesson description.

**Optional Lessons**

As mentioned earlier, not all lessons associated with a unit are part of the standard assignment. They require special instructor action to make them part of the assignment. These lessons are indicated with (curly brackets) in the unit overviews, and with a *Special Note* in the lesson descriptions.

## Notes about Special Units

**Consolidation Units**

Unit 9 is a consolidation unit reviewing the work in addition, subtraction, multiplication, and division of whole numbers found in Units 1 through 8. Students who do not earn credit on a Unit 9 pretest are always stopped. Instructors must review each student's test results and decide which lessons to assign.

Similarly, Unit 16 is a consolidation unit covering the work on fractions, mixed numbers, and decimals found in Units 10 through 15. Students who do not earn credit on a Unit 16 pretest are stopped. Again, instructors must review each student's test results and decide which lessons to assign.

**Multiple Sequences**

As mentioned earlier, assigned lessons appear in sequence, with completed lessons remaining available for review.

Unit 10, however, has three such sequences; when one sequence of lessons is completed, all of those lessons disappear and a second sequence begins, and then the third. See the Unit 10 overview for details.

Similarly, Unit 16 has two such sequences. See the Unit 16 overview for details.

### Contingency Assignment

As mentioned earlier, Unit 12 has one set of lessons assigned on failure of pretest but a different set assigned on failure of posttest. See the Unit 12 overview for details.

## Special Notes

### Lesson Titles and Filenames

Throughout this guide, lesson titles are the titles that appear on the screen when students enter the lesson. Sometimes the titles instructors see through their SYS 4 group catalog will be different from the title page of the lesson. This is due either to space limitations, or to a desire for consistency in SYS 4 names. Where it is different, the SYS 4 catalog name is given in parentheses under the primary lesson title in the lesson description.

Sometimes only parts of lessons are used in this curriculum, and the primary lesson title does not reflect this. In these cases, some discriminating identifier has been added in square brackets within this curriculum guide. For instance, *Facts Practice [Addition]* indicates that only *addition facts* are given.

When only a part of a lesson is being used, or a different version with a different filename is used with the SYS 4 management system, the SYS 4 filename is shown in parentheses under the primary lesson filename.

### List of Lessons

*Appendix B* contains an alphabetical listing of filenames used in this curriculum, with references to the page numbers where descriptions can be found. Sometimes an entire lesson is assigned, and sometimes only a part of it. Details can be seen by examining each description.

## Conclusion

We hope you find the lessons in the *CCA Basic Skills Mathematics Curriculum* and this documentation useful. As always, we invite your comments on the lessons, the curriculum, and the documentation.

You may be interested in other instructor's guides in the basic skills series, or in guides containing curricula for high school students:

*Instructor's Guide to the CCA Basic Skills Reading Comprehenion Curriculum*
*Instructor's Guide to the CCA Basic Skills Language Arts Curriculum*
*Instructor's Guide to the CCA Basic Skills Spelling Curriculum*
*Instructor's Guide to the CCA High School Social Science Curricula: U.S. Government, U.S. History, and Economics*
*Instructor's Guide to the CCA High School Science Curricula: Earth Science, Biology, and Physical Science*

Guides in press:

*Instructor's Guide to the CCA High School Mathematics Curricula: General Mathematics, Pre-Algebra, Algebra I, Algebra II, and Calculus*
*Instructor's Guide to the CCA High School English Curricula: English I - III, Study Skills, and Thinking Skills*

**Acknowledgements**

John Gilpin selected and sequenced the lessons in the *CCA Basic Skills Mathematics Curriculum*. The pre- and posttest items were written by members of the CCA staff. The paper-and-pencil worksheets were edited by Dorothy Silver from materials originally developed by Hank Barnes, classroom instructor at Sheridan Correctional Center, Sheridan, Illinois. Dorothy O'Connell edited this curriculum guide, and Joan Phebus formatted and printed it.

Your instructor must look at your record
before you can continue with this topic.
A message has been sent to your instructor.
For now, press BACK for your topic index.

**Figure 1. Page students see when stopped in the curriculum, waiting for an instructor action.**

Ccc / b: "p.m. class" Remaining Spaces:
Press a to add someone to the roster
d delete someone from the system
r see someone's records
n change someone's name
p change someone's password
SHIFT-NEXT - next section
SHIFT-BACK - previous section
SHIFT-DATA - section optio
LAB - another secti

**Figure 2. Roster page with solid flag indicating a student is waiting for instructor action.**

Current record for student:
1 READING COMPREHENSION
2 MATHEMATICS
3 LANGUAGE ARTS
LESSON Speedway (18 min) 2/1 - 2/2 | 2 | 8:41 | Lesson Completed
TEST Unit 0 Posttest on 2/2 Score - 60% NO CREDIT
NOTE No credit earned on the Unit 0 Posttest. The student is waiting for an instructor action.
l - log
v - view test
p - prescription
r - retake posttest
( - previous unit
) - next unit
g - general info
t - topic summary
1-3 - single topic
c - copy
h - history
a - assign
d - delete
o - order topics
m - min delay
n - note
SHIFT-NEXT - next record
NEXT - next record
BACK - prior record
LAB - roster page
SHIFT-HELP - delete record
indicates student is waiting for an instructor action

**Figure 3. Options available to instructor when student is at a *wait state*.**

**Figure 4. Instructor viewing individual test results.**

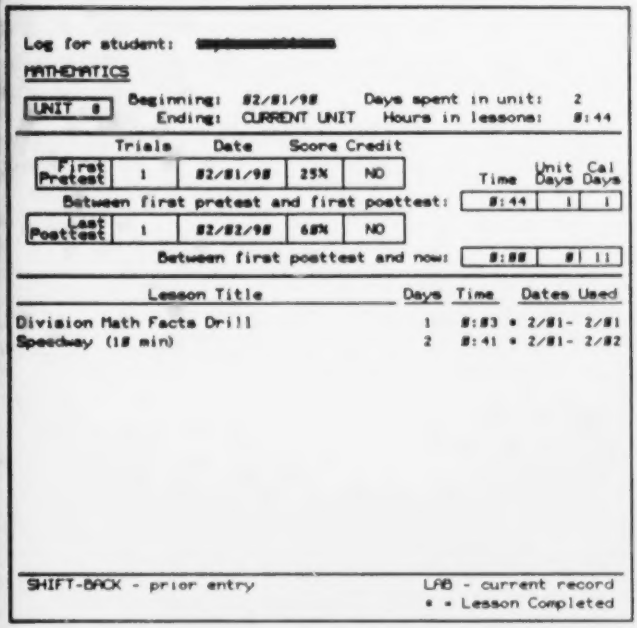

Log for student: [redacted]
MATHEMATICS
UNIT 8
Beginning: 02/01/98
Ending: CURRENT UNIT
Days spent in unit: 2
Hours in lessons: 0:44
Trials Date Score Credit
First Pretest 1 02/01/98 25% NO
Time Unit Days Cal Days
Between first pretest and first posttest: 0:44 1 1
Last Posttest 1 02/02/98 68% NO
Between first posttest and now: 0:00 0 11
Lesson Title Days Time Dates Used
Division Math Facts Drill 1 0:03 * 2/01- 2/01
Speedway (10 min) 2 0:41 * 2/01- 2/02
SHIFT-BACK - prior entry
LAB - current record
* = Lesson Completed

**Figure 5. Instructor viewing record of previous activity by student.**

Lesson prescription for Unit #8:
DO a. Division Math Facts Drill
SKIP b. Long Division 2 (15 min)
DO c. Long Division 3 (20 min)
DO d. Speedway (10 min)
Press a letter to change the lesson's prescription.
SHIFT-HELP - assign above
LAB - student's record
SHIFT-BACK - start over

**Figure 6. Instructor options to customize standard assignments by toggling the SKIP/DO settings.**

**Figure 7. Student sees that lesson *a* has been completed, lesson *b* is ready to work on (lessons *c* and *d* not yet visible).**

# Unit 1

## Basic Addition

**Objective** To add numbers that do not require carrying.

**Description** This unit serves as an introduction to addition. Students learn the concept and process of simple addition without carrying. Also, they learn and practice addition facts.

**Lessons**
- Adding Lengths
  Facts Practice [Addition]

**Offline Materials**
- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | Adding Ones Numbers |
| 3A, | 3B, | 3C | Adding Tens Numbers |
| 5A, | 5B, | 5C | Adding Hundreds Numbers |

**Lesson** **Adding Lengths**

**File** *lengths*

**Authors** Charles S. Weaver
Paul Cohen
William Bonetti

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To write an addition number sentence, given a picture of an addition problem.
- To write an addition number sentence, given a story problem representing an addition problem.

**Description** Students are introduced to a model of addition that views addition as the combining of lengths (see Figure 1).

**Interaction**

- Students are given options of moving on by pressing NEXT or reviewing material by pressing BACK at critical points in the lesson.
- Students are taught to write addition number sentences based on the picture story shown (see Figure 2). The picture story and number sentence are then translated into words (see Figure 3).
- Students write a number story associated with the word problem presented, and then the computer generates a picture of the problem to emphasize the relationship (see Figure 4).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (open-ended, minimum of two minutes required)

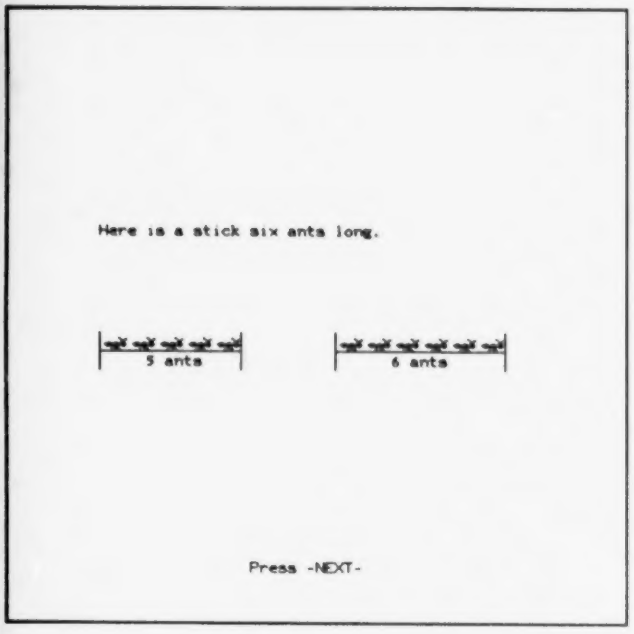



**Figure 1**

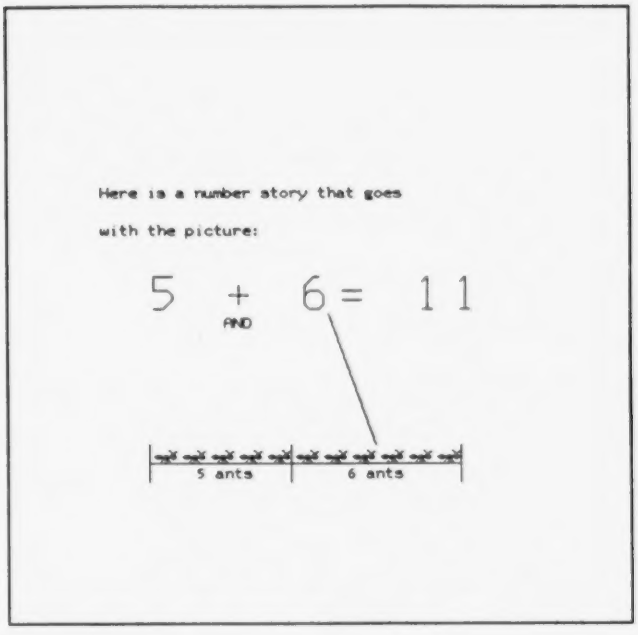



**Figure 2**

Write a number story that goes with the picture.
› 5 + 3 = 8
VERY GOOD, Denise!
Your story says that when you put
a 5 ant stick next to
a 3 ant stick you get
an 8 ant stick.
5 ants
3 ants
8 ants
Press -NEXT- for more.
Press -LAB- to try the number story again.

**Figure 3**

Please write a number story that says
when you put a stick 4 ants long
next to a stick 3 ants long
you get a stick 7 ants long.
› 4 + 3 = 7
4 ants
3 ants
And when we put them together:
7 ants
Press -NEXT- to leave this lesson.
Press -BACK- to try this again.

**Figure 4**

**Lesson** **Facts Practice [Addition]**

(*SYS 4 catalog name*: **Addition Math Facts Drill**)

**File** *facts (SYS 4 file name: factspcp.7)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Helen Leung
Martin A. Siegel
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
and Courseware & Curriculum Applications (CCA) Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To memorize the addition facts, with response time of less than three seconds.

**Description** Students practice addition facts in vertical form. Speed, as well as accuracy, is required for successful completion (see Figure 1).

**Interaction**

- Students work with sets of 10 addition facts, so that groups of facts are practiced and mastered.
- Both commutative forms are given.
- If a fact is answered correctly twice in a row, it is displayed as a mastered fact on the screen.
- If a fact is mastered without errors, the chart is marked with an asterisk.
- A second asterisk is given if the final iteration is answered in less than three seconds.
- If a fact is missed, feedback is given with the correct answer. Then, that fact is returned to the item queue and will recur at increasing intervals. This helps ensure mastery of each fact.
- A display of facts that students have mastered is given after they complete each set of problems (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 25 minutes (self-paced)

**Associated Lessons** *Facts Practice [Subtraction]*
*Speedway! [Add and Subtract]*

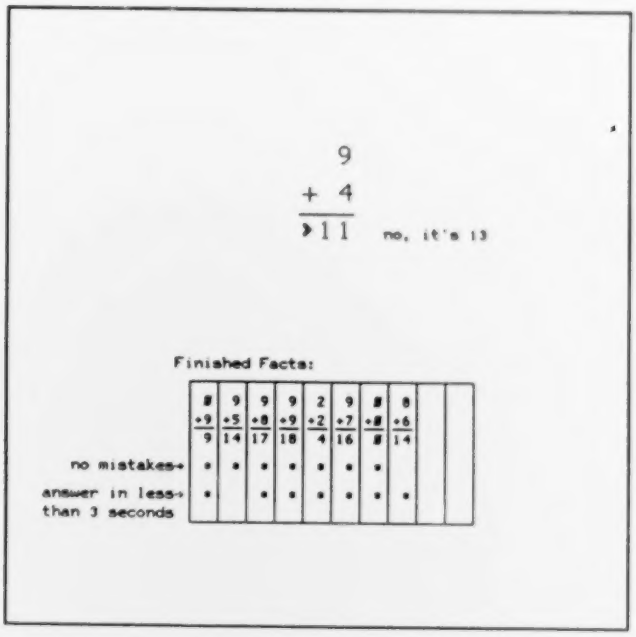



**Figure 1**

denise natour's Addition Facts
You have completed 8 sessions of addition facts practice.
You have mastered 1's, 2's, 3's, 4's, 5's, 6's, and 7's.
Press NEXT to leave
or LAB to do another set of problems.

**Figure 2**

# Unit 2

## Advanced Addition

**Objective** To add numbers that require carrying.

**Description** Students are taught to work addition problems that require carrying. They also continue to learn and practice addition facts.

**Lessons**

- Facts Practice [Addition]
  Adding and Carrying
  Checkup on Addition
  Speedway! [Add and Subtract]

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | Learning to Carry and Carrying to the Tens Column |
| 3A, | 3B, | 3C | Carrying to Hundreds Column |
| 5A, | 5B, | 5C | Carrying to Thousands and Ten-Thousands Column |

**Lesson** **Facts Practice [Addition]**

(*SYS 4 catalog name*: **Addition Math Facts Drill**)

**File** *facts (SYS 4 file name: factspcp.7)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Helen Leung
Martin A. Siegel
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
and Courseware & Curriculum Applications (CCA) Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To memorize addition facts with response time of less than three seconds each.

**Description** This lesson has been described in Unit 1, pages 4 and 5.

**Note**: If this lesson has been successfully completed in Unit 1, it will not be assigned in Unit 2. If students have tested out of Unit 1, and thus have not completed this drill, then it will be assigned.

**Lesson** **Adding and Carrying**

**File** *carry*

**Authors** Esther R. Steinberg
David Leiser

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To separate a two-digit number into groups of ten and groups of ones.
- To add two or three one-digit or two-digit numbers with a carry.

**Description** Students are taught the addition algorithm with carrying. A model of loose sticks (for ones) and bundles of sticks (for 10s) is used. Carrying is accomplished by regrouping 10 loose sticks into one bundle. Students progress from working with pictures to working with addition in standard vertical form.

**Interaction**

- Students work addition problems by associating bundles of 10 sticks and single sticks with numbers, then putting the sticks together and regrouping loose sticks into bundles (see Figure 1).
- Students proceed to work problems written in vertical form with small sticks and bundles above the appropriate digits. Feedback reminds students to carry (see Figure 2).
- Students then demonstrate competency as the traditional addition algorithm is used without pictures of sticks (unless help is required). (See Figure 3.)

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons** *Checkup on Addition*
*Subtraction with Sticks*

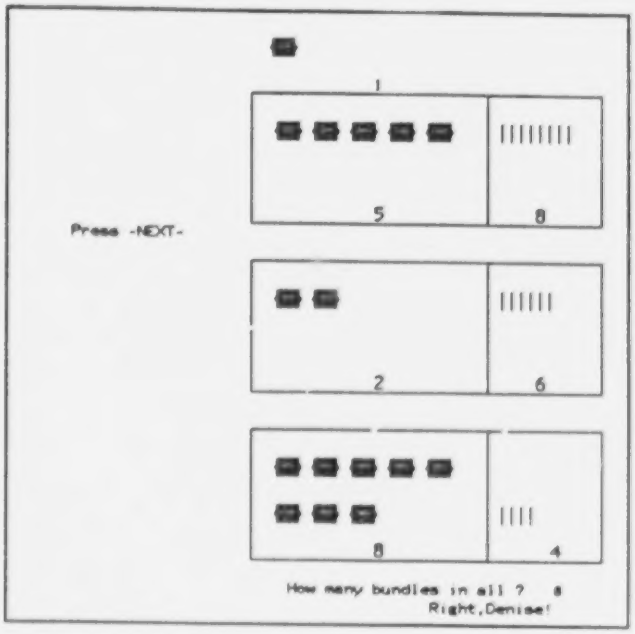



**Figure 1**

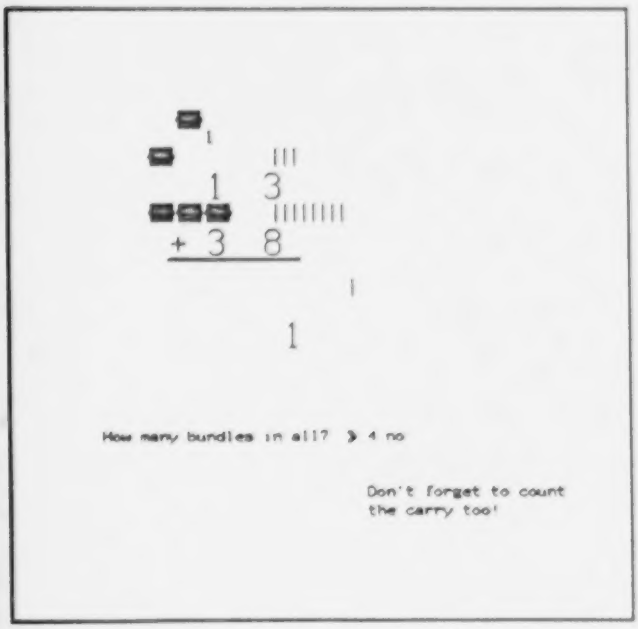



**Figure 2**

**Figure 3**

**Lesson** ## Checkup on Addition

(*SYS 4 catalog name*: **Addition Checkup**)

**File** *addchk*

**Authors** Bonnie Anderson Seiler
Charles S. Weaver
James H. Wilson

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To add two or three two-digit or three-digit numbers with a carry.

**Description** This lesson measures competency in working five addition problems that require carrying. Problems involve the adding of between two and four numbers arranged in vertical format. Numbers vary from two- to four-digit numbers.

To pass the checkup, students must answer all five problems correctly, and at least four must be correct on the first attempt.

**Interaction**

- Students are presented with addition problems in vertical format.
- An option is available to show the number being carried (see Figure 1).
- If students do not meet criteria, they must come back to the lesson and try again.
- Students can choose among six sets of problems.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes

**Associated Lesson** *Adding and Carrying*

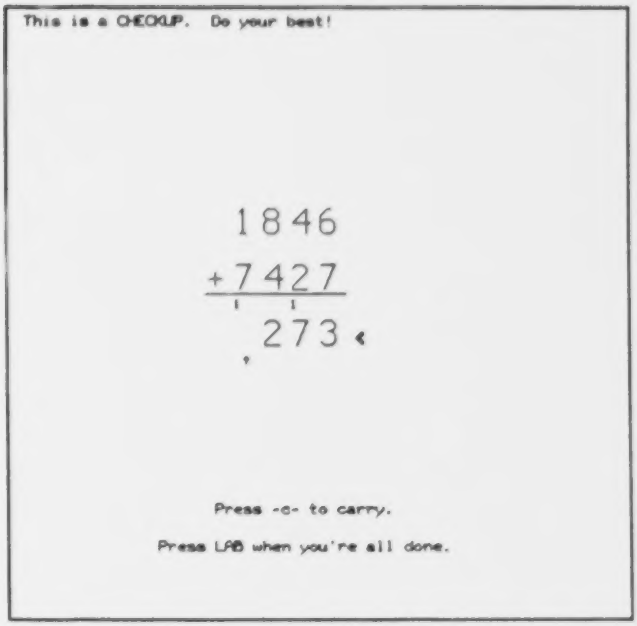

This is a CHECKUP. Do your best!
1846
+7427
1 1
273
Press -c- to carry.
Press LAB when you're all done.

**Figure 1**

**Lesson** ## Speedway! [Add and Subtract]

(*SYS 4 catalog name*: **Addition and Subtraction Speedway**)

**File** *easyspeed*

**Author** Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To increase speed and accuracy in answering addition and subtraction facts.

**Description** This educational game provides drill, practice, and remediation in the basic number facts involving the operations of addition and subtraction. Students enter car races in which they work 10 problems at a time and attempt to beat their own individual race speeds. The cars move according to how fast the students answer problems. Winning is based upon self-improvement.

Students select the operation they wish to work on. However, if students repeatedly practice one operation to the exclusion of areas in which they need work, the lesson limits their choices.

Students first complete one *practice lap*. The time required for that lap becomes the time used by their *opponent's* car to complete the next lap. On successive laps, students can choose whether to race against their most recent lap or their fastest lap.

Note: This is the same lesson as described in Unit 5, pages 36 through 38, except that only addition and subtraction problems are given here.

**Interaction**

- HELP, in the form of pictures for the problems, is always available. This same help is automatic after three failed attempts on a given problem (see Figure 1).
- If HELP is chosen, the timer continues to run, and credit is not given for that problem.
- Response feedback is "OK," "Too small," or "Too large."
- Missed problems are remembered (see Figure 2) and recycled; that is, they are brought back for further practice in subsequent races to ensure mastery.

- Graphs and charts showing performance are available after each game (see Figure 3).
- Students have a choice of doing only addition, only subtraction, or a combination of the two operations.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (open-ended, minimum of 10 minutes required)

**Associated Lessons** *Facts Practice [Addition]*
*Facts Practice [Subtraction]*

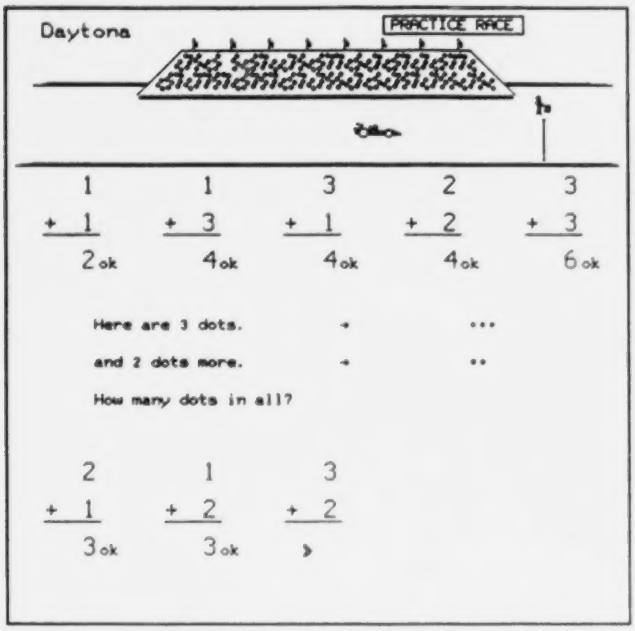



**Figure 1**

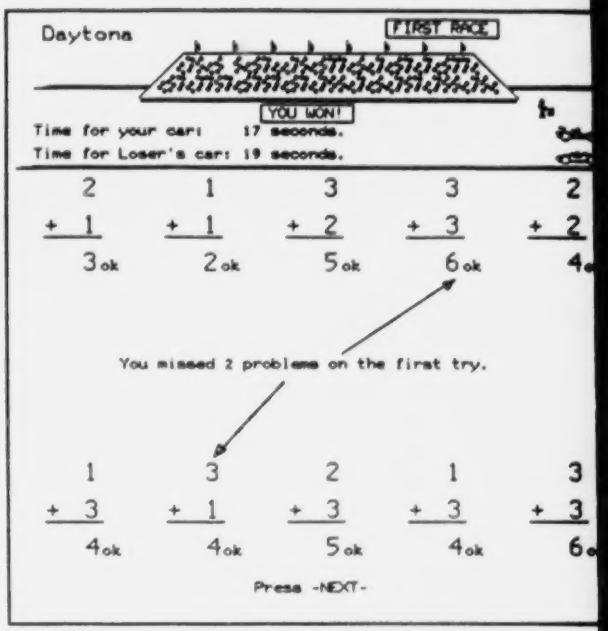

Daytona
FIRST RACE
YOU WON!
Time for your car: 17 seconds.
Time for Loser's car: 19 seconds.
2 1 3 3 2
+ 1 + 1 + 2 + 3 + 2
3ok 2ok 5ok 6ok
You missed 2 problems on the first try.
1 3 2 1 3
+ 3 + 1 + 3 + 3 + 3
4ok 4ok 5ok 4ok
Press -NEXT-

**Figure 2**

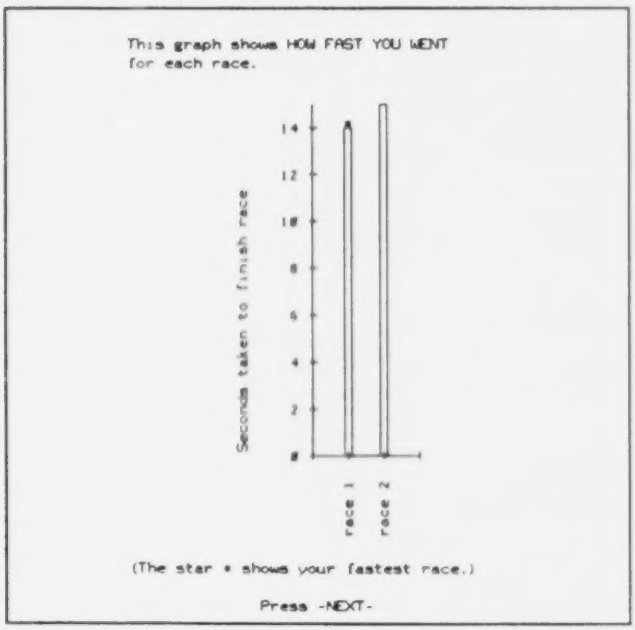



**Figure 3**

# Unit 3

## Subtracting without Borrowing

**Objective** To subtract numbers that do not require borrowing.

**Description** Students learn and practice subtraction facts in this unit.

**Lessons**
- Facts Practice [Subtraction]
  Speedway! [Add and Subtract]

**Offline Materials**
- CCA Assignment Sheet
- CCA Worksheets

| | |
|---|---|
| 1A, 1B, 1C | Learning to Subtract |
| 3A, 3B, 3C | Subtracting Tens Numbers |
| 5A, 5B, 5C | Subtracting Larger Numbers and Zero Has a Place |

**Lesson** **Facts Practice [Subtraction]**

(*SYS 4 catalog name*: **Subtraction Math Facts Drill**)

**File** *facts (SYS 4 file name: factspcp.8)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Helen Leung
Martin A. Siegel
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
and Courseware & Curriculum Applications (CCA) Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To memorize the subtraction facts, with response time of less than three seconds each.

**Description** Students practice subtraction facts in vertical form. Speed, as well as accuracy, is required for successful completion (see Figure 1).

**Interaction**

- Students work with sets of 10 subtraction facts, so that groups of facts are practiced and mastered.
- Both commutative forms are given.
- If a fact is answered correctly twice in a row, it is displayed as a mastered fact on the screen.
- If a fact is mastered without errors, the chart is marked with an asterisk.
- A second asterisk is given if the final iteration is answered in less than three seconds.
- If a fact is missed, feedback is given with the correct answer. Then, that fact is returned to the item queue and will recur at increasing intervals. This helps ensure mastery of each fact.
- A display of facts that the students have mastered is given after they complete each set of problems (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 25 minutes (self-paced)

**Associated Lessons** *Facts Practice [Addition]*
*Speedway! [Add and Subtract]*

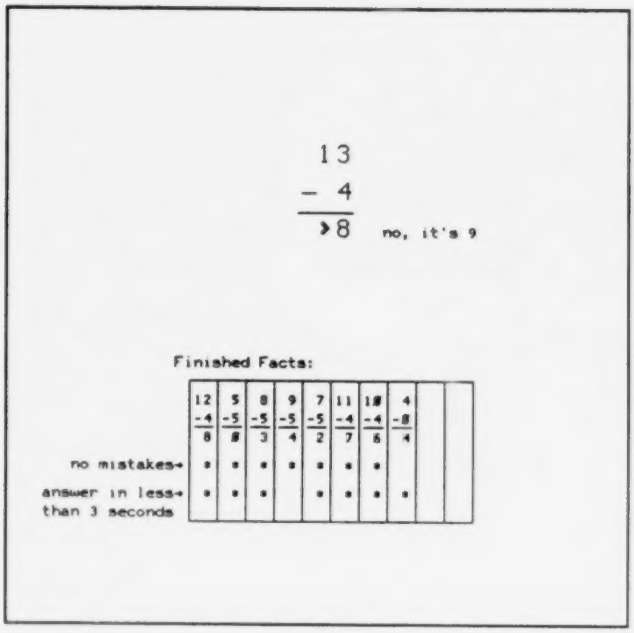



**Figure 1**

denise natour's Subtraction Facts
You have completed 5 sessions of subtraction facts practice.
You have mastered 1's, 2's, and 3's.
Press NEXT to leave
or LAB to do another set of problems.

**Figure 2**

**Lesson** **Speedway! [Add and Subtract]**

(*SYS 4 catalog name*: **Addition and Subtraction Speedway**)

**File** *easyspeed*

**Author** Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To increase speed and accuracy in answering addition and subtraction facts.

**Description** This lesson has been described in Unit 2, pages 12 through 14.

# Unit 4

## Subtracting with Borrowing

**Objective** To subtract numbers that require borrowing.

**Description** Students work subtraction problems that require borrowing. They also continue to learn and practice subtraction facts.

**Lessons**
- Subtraction with Sticks
  Facts Practice [Subtraction]
  {Torpedo (Whole Numbers)}

**Offline Materials**
- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | Learning to Borrow |
| 3A, | 3B, | 3C | Borrowing from Tens Column |
| 5A, | 5B, | 5C | Borrowing from Thousands and Ten-Thousands Column |
| 7A, | 7B, | 7C | Borrowing from More Than One Column |
| 9A, | 9B, | 9C | Borrowing from Zero |

{ } Lesson will not be assigned unless the instructor chooses to do so. (See Preface for explanation.)

**Lesson** **Subtraction with Sticks**

**File** *sticksv*

**Authors** Charles S. Weaver
Bonnie Anderson Seiler
Esther R. Steinberg
William Bonetti

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To subtract two-digit numbers with borrowing.

**Description** Students are taught the subtraction algorithm with borrowing. A model of loose sticks (for ones) and bundles of sticks (for 10s) is used. Borrowing is accomplished by opening a bundle. Students take care of regrouping and the bookkeeping involved with borrowing. Students progress from working with pictures to working with subtraction in standard vertical form.

**Interaction**

- Subtraction problems are given in vertical format. The minuend is shown as bundles and loose sticks (see Figure 1).
- Students press the *b* key to indicate they wish to borrow from the *tens place*. They are then prompted to do the necessary bookkeeping and subtraction (see Figure 2).
- If students are unable to get the correct answer, then sticks and leading questions are provided to get them on track (see Figure 3).
- After students have practiced with the stick model, the sticks are dropped, and they are presented with problems in standard vertical form (see Figure 4).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (open-ended, minimum of three minutes required)

**Associated Lessons** *Facts Practice [Subtraction]*
*Adding and Carrying*

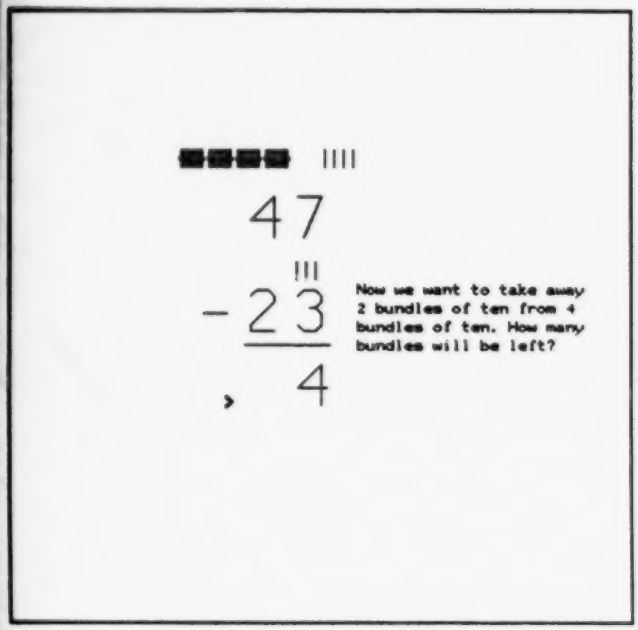



**Figure 1**

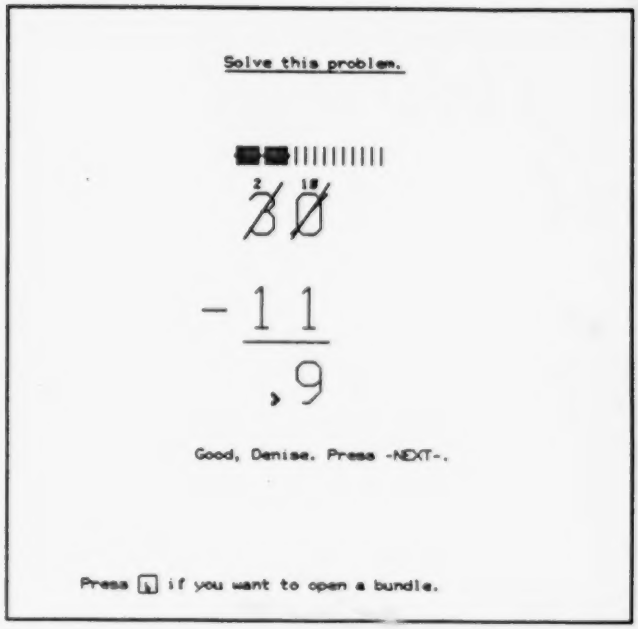



**Figure 2**

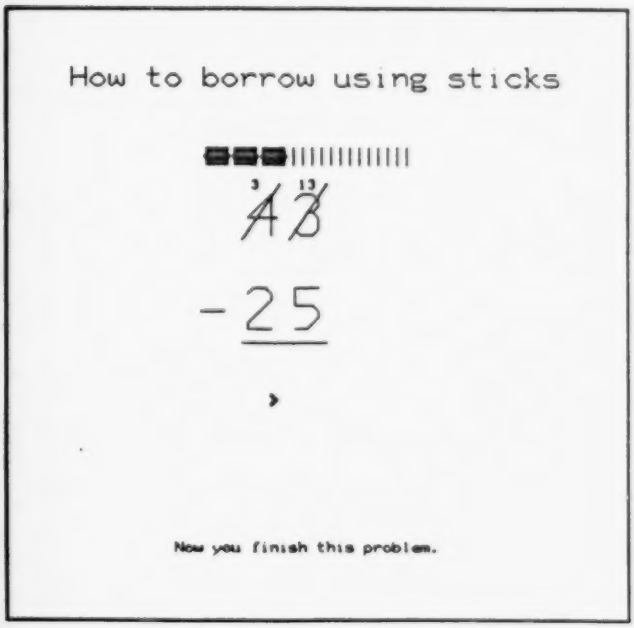



**Figure 3**

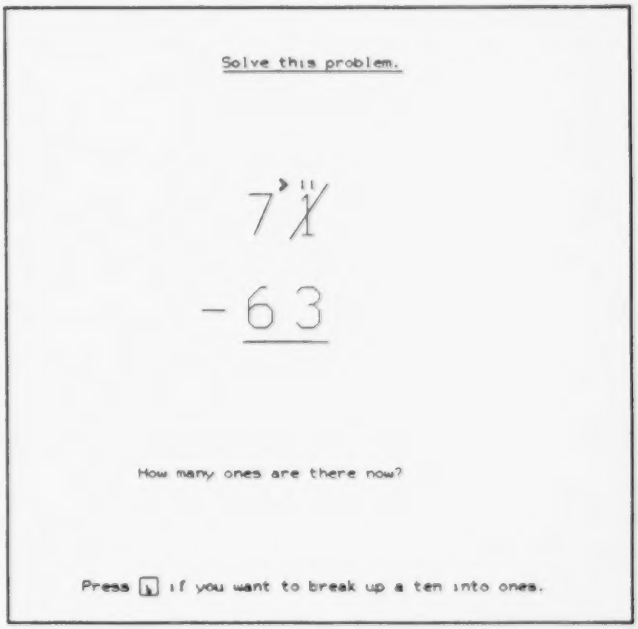



**Figure 4**

**Lesson** ## Facts Practice [Subtraction]

(*SYS 4 catalog name*: **Subtraction Math Facts Drill**)

**File** *facts (SYS 4 file name: factspcp.8)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Helen Leung
Martin A. Siegel
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
and Courseware & Curriculum Applications (CCA) Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To memorize subtraction facts.

**Description** This lesson has been described in Unit 3, pages 16 and 17.

**Note:** If this lesson has been successfully completed in Unit 3, it will not be reassigned in Unit 4. If students have tested out of Unit 3, and thus have not completed this drill, then it will be assigned.

**Lesson** **Torpedo [Whole Numbers]**

**File** *sea (SYS 4 filename: sea.6)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliattion** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To estimate whole number distances on a number line.

**Description** Students are provided with a game situation in which they estimate relative distances on a number line. One player has a submarine; the other, a boat. The object of the game is to move to a point above (or below) their opponent's piece and shoot it (see Figure 1).

**Interaction**

- Students are provided with a list of potential opponents (others on the system and in the same lesson), or they may choose to play against the computer.
- If a request is made to move out of range, then students are told the move is out of range and may try again.
- Obstacles (fish and other sea life) block the way at various points along the number line. Obstacles provide points of reference and block shots, thus increasing the need for accuracy (see Figure 2).
- The range on the number line varies providing the students with different perspectives and challenges.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (open-ended, minimum of 10 minutes required)

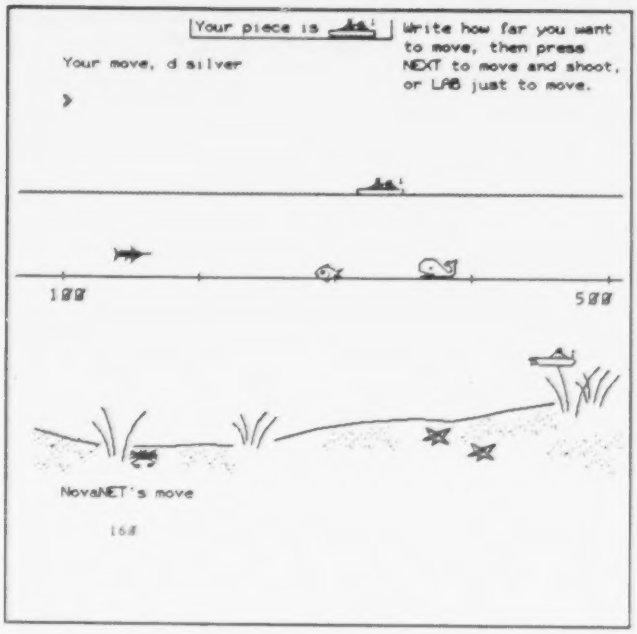

Your piece is
Write how far you want to move, then press NEXT to move and shoot, or LAB just to move.
Your move, d silver
>
100
500
NovaNET's move
168

**Figure 1**

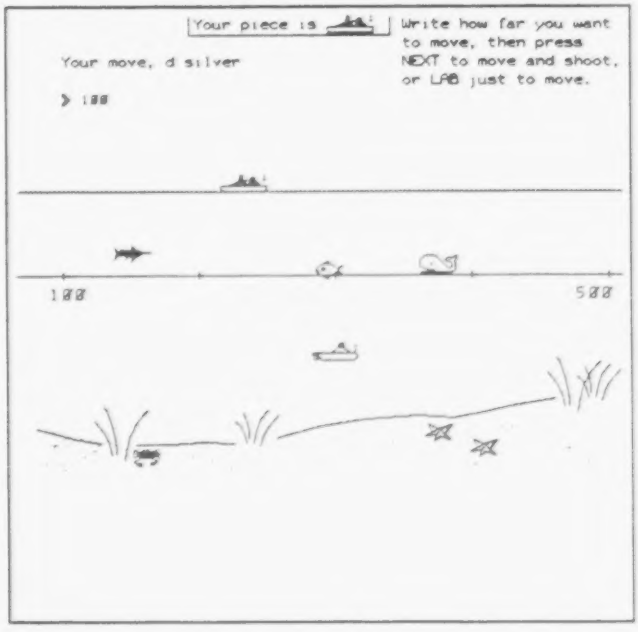

Your piece is
Write how far you want to move, then press NEXT to move and shoot, or LAB just to move.
Your move, d silver
> 100
100
500

**Figure 2**

# Unit 5

# Learning to Multiply

**Objective** To multiply by single-digit numbers.

**Description** This unit serves as an introduction to multiplication. Students are taught the concept and process of multiplication. Also, they learn and practice multiplication facts.

**Lessons**

- Facts Practice [Multiplication]
  (Rubber Stamp)
  (Arrays)
  Multiplying by a One-Digit Number
  Multiplication Is ...
  (Speedway! [Add, Subtract, Multiply, Divide])

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | |
|---|---|
| 1A, 1B, 1C | Learning to Multiply |
| 3A, 3B, 3C | Multiplying Tens Numbers |
| 5A, 5B, 5C | Carrying to the Tens Column |
| 7A, 7B, 7C | Multiplying Ones, Tens, and Hundreds |
| 9A, 9B, 9C | Carrying to the Hundreds Column |
| 11A, 11B, 11C | Multiplying Thousands Numbers (No Carrying) |
| 13A, 13B, 13C | Carrying to the Ten-Thousands Column |

---

( ) Lesson will not be assigned unless the instructor chooses to do so. (See Preface for explanation.)

**Lesson** **Facts Practice [Multiplication]**

(*SYS 4 catalog name*: **Multiplication Math Facts Drill**)

**File** *facts (SYS 4 file name: factspcp.9)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Helen Leung
Martin A. Siegel
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
and Courseware & Curriculum Applications (CCA) Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To memorize the multiplication facts, with response time of less than three seconds each.

**Description** Students practice multiplication facts in vertical form. Speed, as well as accuracy, is required for successful completion (see Figure 1).

**Interaction**

- Students work with sets of 10 multiplication facts, so that groups of facts are practiced and mastered.
- Both forms of commutative facts are given.
- If a fact is answered correctly twice in a row, it is displayed as a mastered fact on the screen.
- If a fact is mastered without errors, the chart is marked with an asterisk.
- A second asterisk is given if the final iteration is answered in less than three seconds.
- If a fact is missed, feedback is given with the correct answer. Then, that fact is returned to the item queue and will recur at increasing intervals. This helps ensure mastery of each fact.
- A display of the facts students have mastered is given after they complete each set of problems (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 25 minutes (self-paced)

**Associated Lessons**
*Facts Practice [Addition]*
*Facts Practice [Subtraction]*
*Facts Practice [Division]*
*Speedway! [Add, Subtract, Multiply, Divide]*

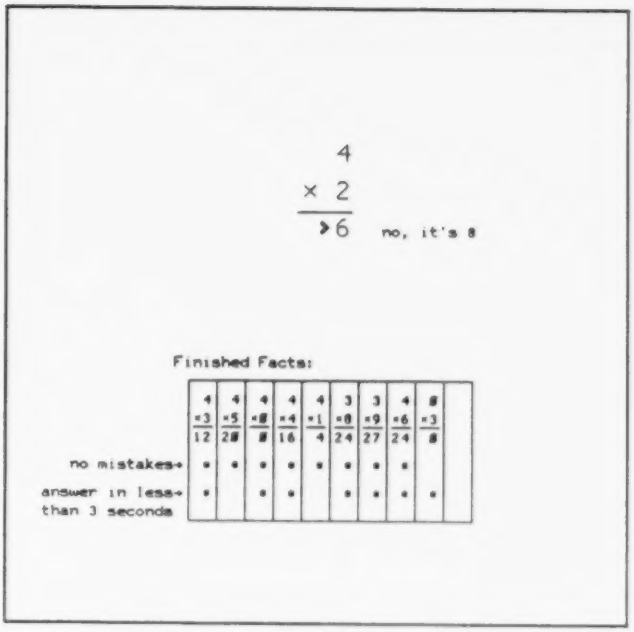



**Figure 1**

denise natour's Multiplication Facts
You have completed 4 sessions of multiplication facts practice.
You have mastered 1's, 2's, and 3's.
Press NEXT to leave
or LAB to do another set of problems.

**Figure 2**

**Lesson** **Rubber Stamp**

**File** *stamp*

**Authors** David Leiser
Brand Fortner
Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives** To write number sentences describing given pictures.

To create pictures representing given number sentences.

**Description** Students *stamp* the screen repeatedly to produce the required number of stamps on their electronic paper. Students then tell how many dots they stamped altogether. Then, a word sentence that describes this picture is shown. Finally, an addition number sentence is shown that describes the picture, and students provide the correct answer to the number sentence (see Figure 1).

Later, a number sentence is given, and students are asked to make a picture representing it.

**Interaction**

- Students can ERASE and rewrite stamps as they wish until they provide the correct number. They are told if they have too many (and have to ERASE) or too few (and have to make more).
- If students write the wrong number sentence, they are asked to look at the picture and try again.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (open-ended, minimum of three minutes required)

**Associated Lessons** *Arrays*
*More Arrays*

**Special Notes**

- Touch capability required
- This lesson will not be assigned unless the instructor chooses to do so.

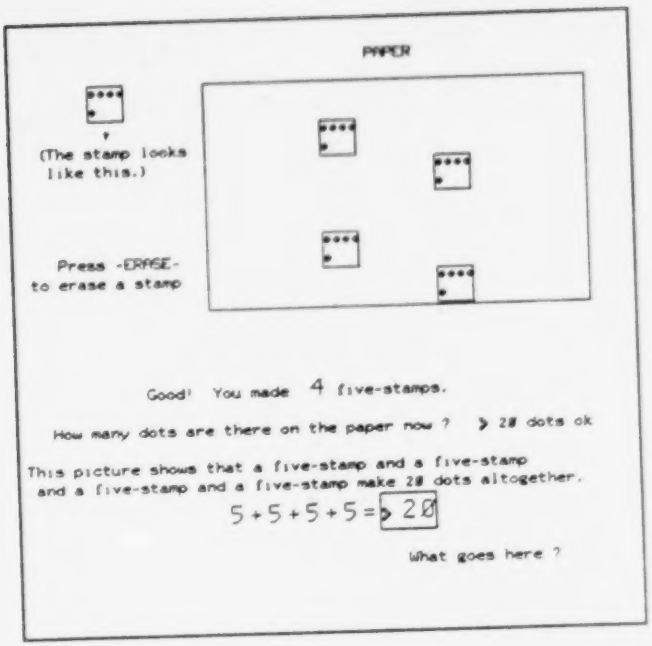

PAPER
(The stamp looks like this.)
Press -ERASE- to erase a stamp
Good! You made 4 five-stamps.
How many dots are there on the paper now ? > 20 dots ok
This picture shows that a five-stamp and a five-stamp and a five-stamp and a five-stamp make 20 dots altogether.
5+5+5+5=> 20
What goes here ?

**Figure 1**

**Lesson** **Arrays**

**File** *arrays*

**Author** Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To relate multiplication number sentences to an array of dots.

**Description** Students see dots grouped and are asked to write a number sentence describing the groupings. The commutative properties of these groupings are shown, without using the term (see Figures 1 and 2).

**Interaction**

- Students are shown an array of dots and asked to guess how many dots there are.
- Students then count the dots and compare the answer with their guess.
- Students are led to write multiplication sentences describing the array of dots.
- If errors are made, feedback is given leading students to the correct answer.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Rubber Stamp*
*More Arrays*

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

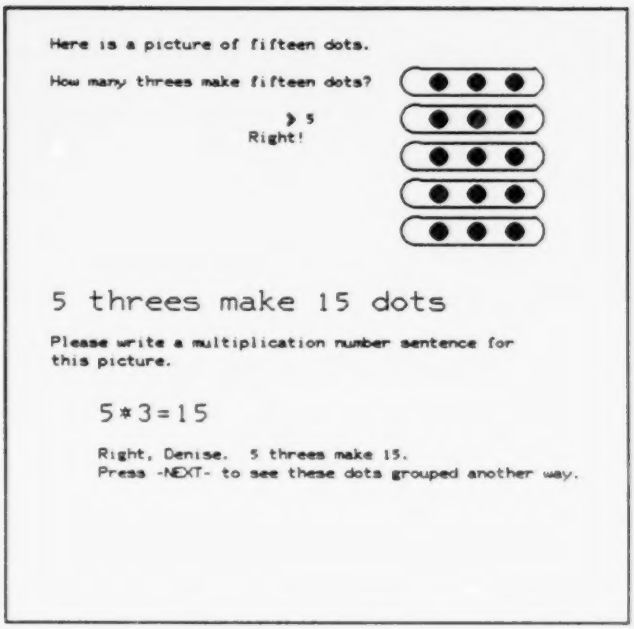

Here is a picture of fifteen dots.
How many threes make fifteen dots?
> 5
Right!
5 threes make 15 dots
Please write a multiplication number sentence for this picture.
5*3=15
Right, Denise. 5 threes make 15.
Press -NEXT- to see these dots grouped another way.

**Figure 1**

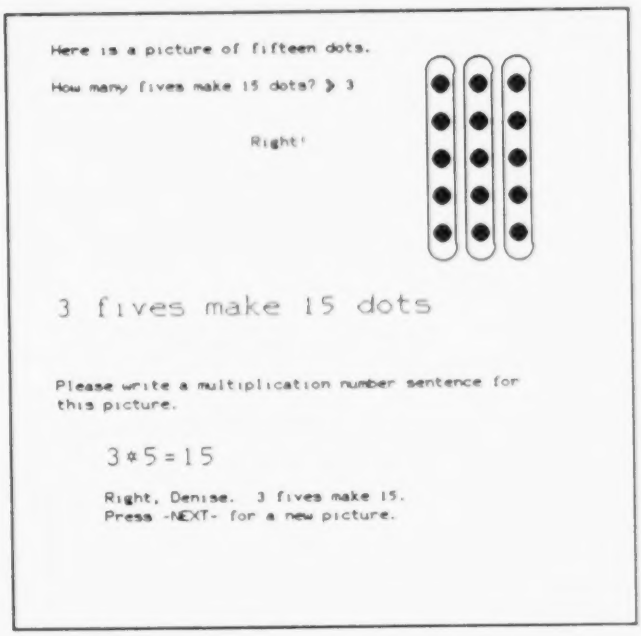

Here is a picture of fifteen dots.
How many fives make 15 dots? > 3
Right!
3 fives make 15 dots
Please write a multiplication number sentence for this picture.
3*5=15
Right, Denise. 3 fives make 15.
Press -NEXT- for a new picture.

**Figure 2**

**Lesson** **Multiplying by a One-Digit Number**

(*SYS 4 catalog name*: **Single-Digit Multiplication** )

**File** *multa1*

**Authors** Charles S. Weaver
Bonnie Anderson Seiler
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To see that multiplication is a short form of repeated addition of a number.

**Description** Students are taught the algorithm for multiplication in terms of repeated addition of the same number.

**Interaction**

- Students are led through the addition of a column of numbers. They are prompted to carry when needed (see Figure 1).
- If students make errors, they are led to the correct answer (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lesson** *Multiplication Is ...*

**Figure 1**

**Figure 2**

**Lesson** **Multiplication Is ...**

**File** *multmeans*

**Authors** Bonnie A. Seiler
Charles Arthur
Marc Miller

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To show students that multiplication and addition are related functions.

**Description** This lesson shows that multiplication is a form of repeated addition (see Figure 1). Students write a number sentence to determine the number of objects in a collection of equal numbered subsets (or the number of objects in an array).

**Interaction**

- Objects are represented on the screen in equal-sized groupings.
- Students write a number sentence describing the picture, first in terms of multiple addition; then, in terms of multiplication.
- If students answer incorrectly three times in a row, they are given the correct answer.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lesson** *Multiplying by a One-Digit Number*

8+8+8+8+8= 40 great
This says that "5 eights added together make 40."
5×8= 40
This MULTIPLICATION NUMBER SENTENCE says that "5 times 8 make 40."
A multiplication number sentence is a faster way to write an addition number sentence that adds the same number.
press -NEXT-

**Figure 1**

**Lesson** **Speedway! [Add, Subtract, Multiply, Divide]**

**File** *speedway*

**Author** Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To enhance speed and accuracy in answering math facts.

**Description** This educational game provides drill, practice, and remediation in learning the basic math facts. Students enter car races, in which they work 10 problems at a time and attempt to beat their own individual race speeds. The cars move according to how fast the students answer problems. Winning is based upon self-improvement.

Students select the operation they wish to work on. However, if students repeatedly practice one operation to the exclusion of areas in which they need work, the system limits their choices.

Students first complete one *practice lap*. The time required for that lap becomes the time required for their *opponent's* car to complete the next lap. On successive laps, students can choose whether to race against their most recent lap, or their fastest lap, or set the time themselves.

Note: This is the same lesson as described in Unit 2, pages 12 through 14, except this lesson uses all four of the operators: addition, subtraction, multiplication, and division.

**Interaction**

- When facts are mastered at one level, students advance to more difficult sets of facts.
- HELP, in the form of pictures for the problems, is always available. This same help is automatic after three failed attempts for each problem (see Figure 1).
- If HELP is chosen, the timer continues to run, and credit is not given for that problem.
- Response feedback is "OK," "Too small," or "Too large."
- Missed problems are remembered (see Figure 2) and recycled; that is, they are brought back for further practice in subsequent races to ensure mastery (see Figure 3).
- Graphs and charts record students' performance and are available after each game to illustrate progress (see Figure 4).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (open-ended, minimum of 10 minutes required)

**Associated Lessons**
*Facts Practice [Addition]*
*Facts Practice [Subtraction]*
*Facts Practice [Multiplication]*
*Facts Practice [Division]*

**Special Note** This lesson will not be assigned unless the instructor chooses to do so. However, it will be automatically assigned in Unit 6 (if students need work in Unit 6).

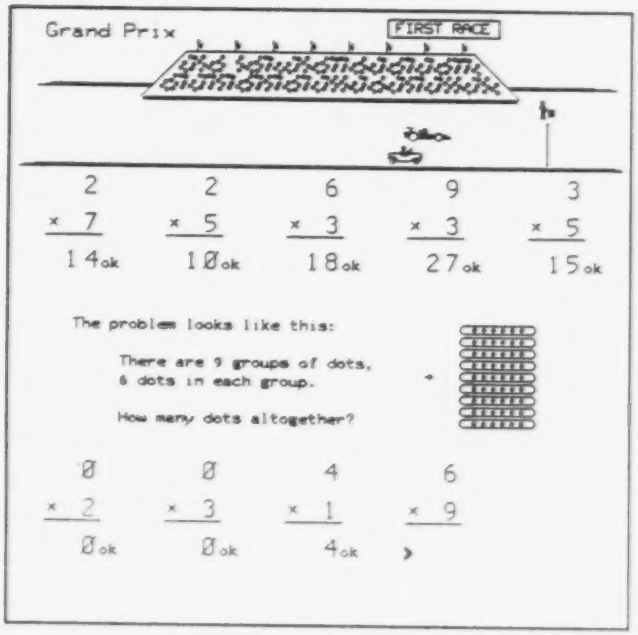

Grand Prix
FIRST RACE
2 × 7 = 14 ok
2 × 5 = 10 ok
6 × 3 = 18 ok
9 × 3 = 27 ok
3 × 5 = 15 ok
The problem looks like this:
There are 9 groups of dots,
6 dots in each group.
How many dots altogether?
0 × 2 = 0 ok
0 × 3 = 0 ok
4 × 1 = 4 ok
6 × 9

**Figure 1**

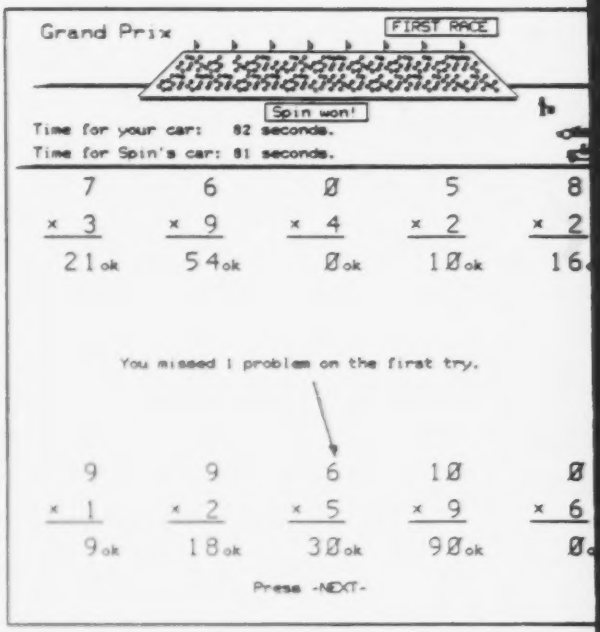

Grand Prix
FIRST RACE
Spin won!
Time for your car: 82 seconds.
Time for Spin's car: 81 seconds.
7 × 3 = 21 ok
6 × 9 = 54 ok
0 × 4 = 0 ok
5 × 2 = 10 ok
8 × 2
You missed 1 problem on the first try.
9 × 1 = 9 ok
9 × 2 = 18 ok
6 × 5 = 30 ok
10 × 9 = 90 ok
0 × 6
Press -NEXT-

**Figure 2**

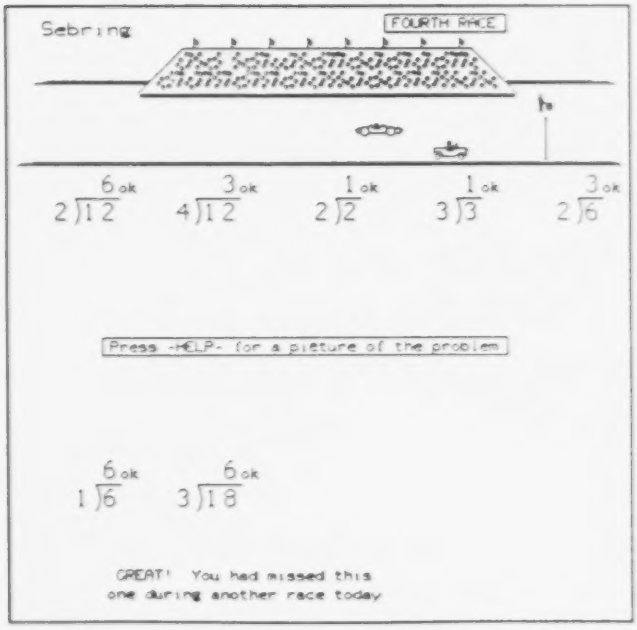



**Figure 3**

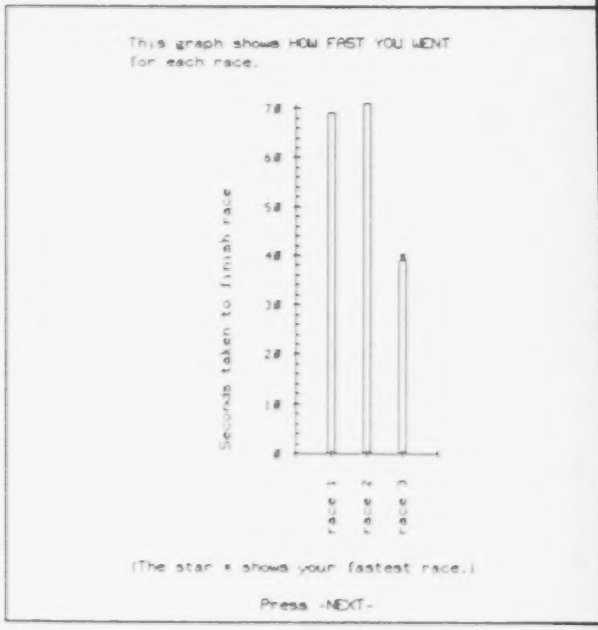



**Figure 4**

# Unit 6

## Multiplying by Two or More Numbers

**Objective** To multiply two numbers that are two and three digits long.

**Description** Students work multiplication problems in which the multiplier has more than one digit, and they continue to learn and practice multiplication facts.

**Lessons**

- Facts Practice [Multiplication]
  {More Arrays}
  {Ant War!}
  Multiplying by Numbers Ending in Zero
  Multiplying by Two-Digit Numbers
  Speedway! [Add, Subtract, Multiply, Divide]

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | Multiplying by Tens Numbers (2-digit by 2-digit) |
| 3A, | 3B, | 3C | Multiplying by Hundreds Numbers (3-digit by 2-digit) |
| 5A, | 5B, | 5C | Multiplying by Hundreds Numbers (4-digit by 2-digit) |
| 7A, | 7B, | 7C | Multiplying by Thousands Numbers (4-digit by 3-digit) |

{ } Lesson will not be assigned unless the instructor chooses to do so. (See Preface for explanation.)

**Lesson** ## Facts Practice [Multiplication]

(*SYS 4 catalog name*: **Multiplication Math Facts Drill**)

**File** *facts (SYS 4 file name: factspcp.9)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Helen Leung
Martin A. Siegel
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
and Courseware & Curriculum Applications (CCA) Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To memorize multiplication facts, with response time of less than three seconds each.

**Description** This lesson has been described in Unit 5, pages 26 and 27.

**Note:** If this lesson has been successfully completed in Unit 5, it will not be assigned in Unit 6. If students have tested out of Unit 5, and thus have not completed this drill, then it will be assigned.

**Lesson** **More Arrays**

**File** *arrays2*

**Author** Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To illustrate multiplication problems by building two-dimensional arrays.

**Description** Students touch the screen to build two-dimensional arrays representing multiplication number sentences (see Figure 1).

**Interaction**
- Given a number sentence, students build an array to describe that sentence.
- Appropriate feedback for incomplete arrays or wrong-sized arrays is given, which leads students to the correct answer (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Rubber Stamp*
*Arrays*

**Special Notes**
- Touch capability required
- This lesson will not be assigned unless the instructor chooses to do so.

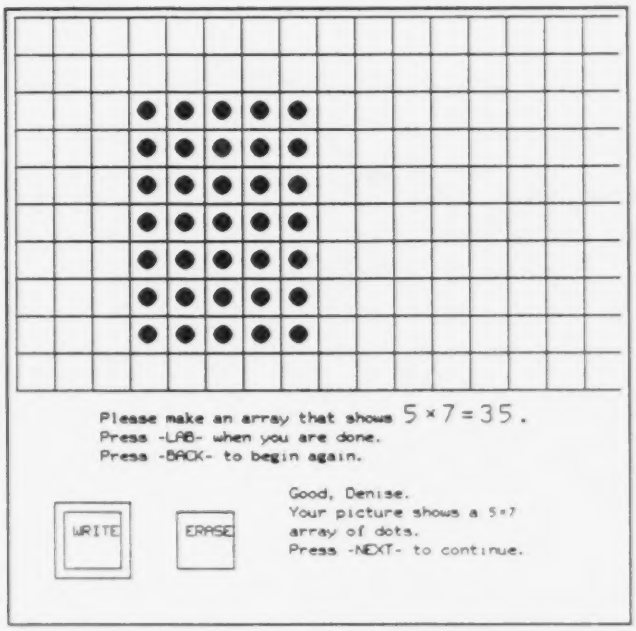

Please make an array that shows 5 × 7 = 35.
Press -LAB- when you are done.
Press -BACK- to begin again.
WRITE
ERASE
Good, Denise.
Your picture shows a 5×7
array of dots.
Press -NEXT- to continue.

**Figure 1**

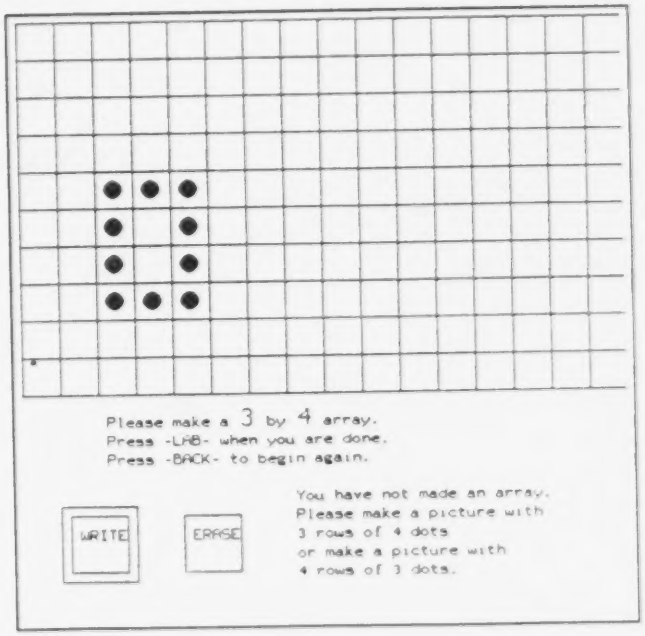

Please make a 3 by 4 array.
Press -LAB- when you are done.
Press -BACK- to begin again.
WRITE
ERASE
You have not made an array.
Please make a picture with
3 rows of 4 dots
or make a picture with
4 rows of 3 dots.

**Figure 2**

**Lesson** **Ant War!**

**File** *antwar*

**Authors** Bonnie Anderson Seiler
Brand Fortner

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To answer word problems involving arrays, comparisons, and sums.

**Description** Students work story problems in a game situation. The object of the game is to lead the army of red ants to victory in at least three out of five battles against the black ants. To accomplish this, students must correctly solve the word problems.

**Interaction**

- Students are given spy reports, in word problem form, that describe the size of the opponent's battle force (see Figure 1).
- Students are asked how many ants they want to send into battle. They need at least one more soldier than the opposing army has in order to win the battle (see Figures 2 and 3).
- There are 10 types of problems, and, if a problem type is missed, then it will come up again in the next war.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (open-ended, minimum of five minutes required)

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

BLACK ANT ARMY has
346 soldiers left
RED ANT ARMY has
351 soldiers left
BLACK ANTS' BATTLE PLAN:
Your spy ants have found out that
there will be 7 rows of 5 ants and 35 ants more.
How many Red Ants do you want
to send into this battle? 7
Press -STOP- to forfeit the whole war

**Figure 1**

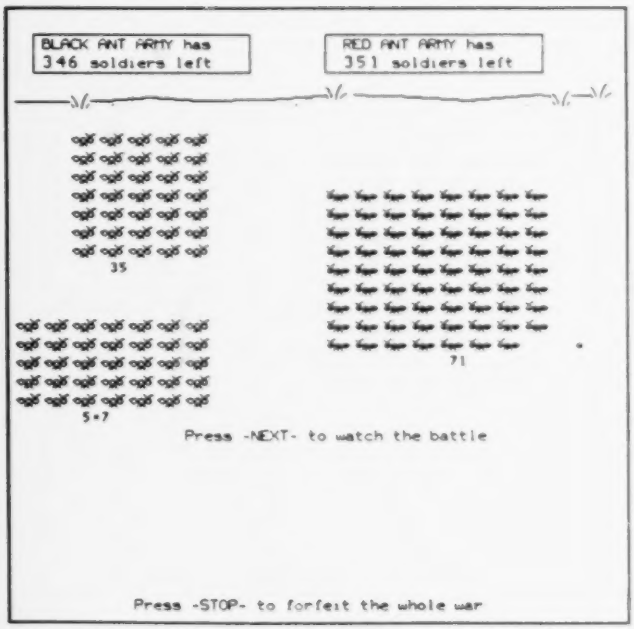



**Figure 2**

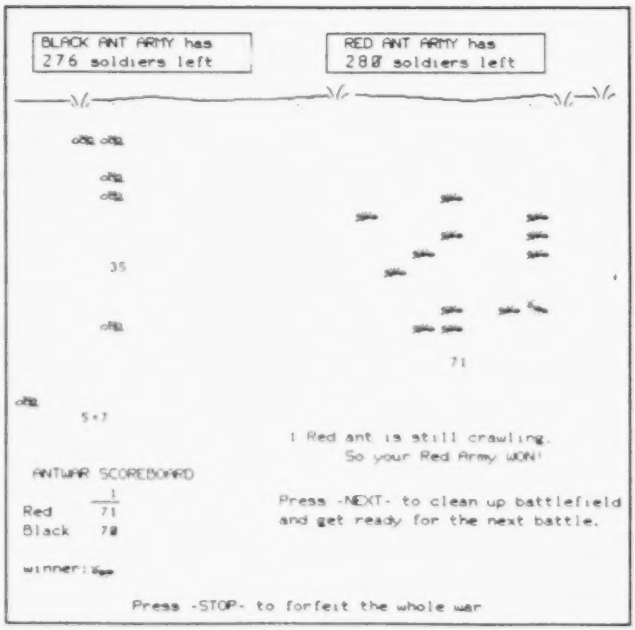



**Figure 3**

**Lesson** ## Multiplying by Numbers Ending in Zero

(*SYS 4 catalog name*: **Multiplying by 10s and 100s**)

**File** *multa2*

**Authors** Bonnie Anderson Seiler
Charles S. Weaver
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To multiply by numbers ending in one or two zeroes.

**Description** Students perform the standard multiplication algorithm with the nonzero digits in the number, and then tack on the necessary number of zeroes.

**Interaction**

- Students rewrite the multiplier that contains the zeroes.
- Students perform the multiplication algorithm on the nonzero part of the number, and then tack on the appropriate number of zeroes (see Figure 1).
- Later in the lesson, students are taught to do the problems in standard form (see Figure 2).
- If errors are made, students are led to the correct answer.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (open-ended, minimum of five minutes required)

Solve this problem by yourself the same way we've been doing other ones.
6 × 900
means the same as
6 × 9 hundreds
which means the same as
54 hundreds
which gives us
› 5400
no
How many is 54×100?

**Figure 1**

When we write the answer to a problem like this one, we usually start at the ONES place ...and work RIGHT to LEFT.
21
× 30
›
Tack on a zero
Let's "tack on" the zero FIRST! Then work the problem 21×3.
Now let's do this same problem the same way, but we will write it like this → 21 × 30

**Figure 2**

**Lesson** **Multiplying by Two-Digit Numbers**

(*SYS 4 catalog name*: **Two-Digit Multiplication**)

**File** *multalg*

**Authors** Charles S Weaver
Bonnie Anderson Seiler
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
and Courseware & Curriculum Applications (CCA) Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To multiply two two-digit numbers.

**Description** Students learn the multiplication algorithm by breaking the problem into a sum of multiplication problems each having one nonzero digit.

**Interaction**

- Students break the problem up into subproblems that they know how to solve (see Figures 1 through 3).
- Students are taught the shorthand representation of the problem (without writing the auxiliary problems). (See Figure 4.)
- Students practice the technique.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (open-ended, minimum of five minutes required)

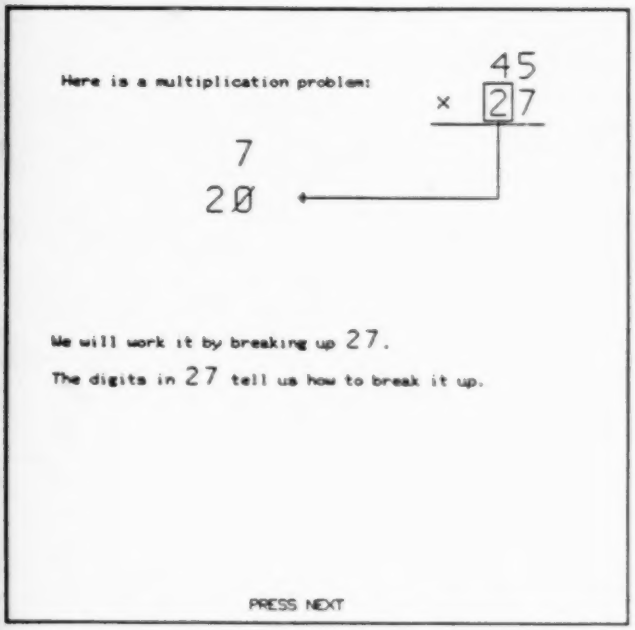



**Figure 1**

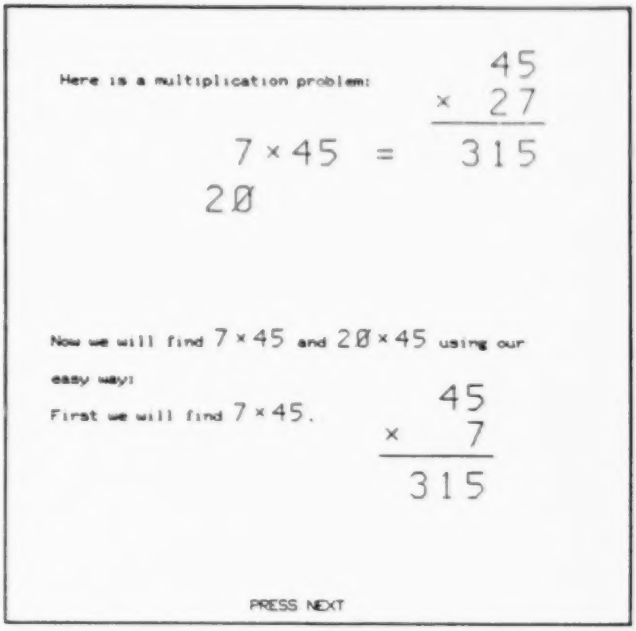



**Figure 2**

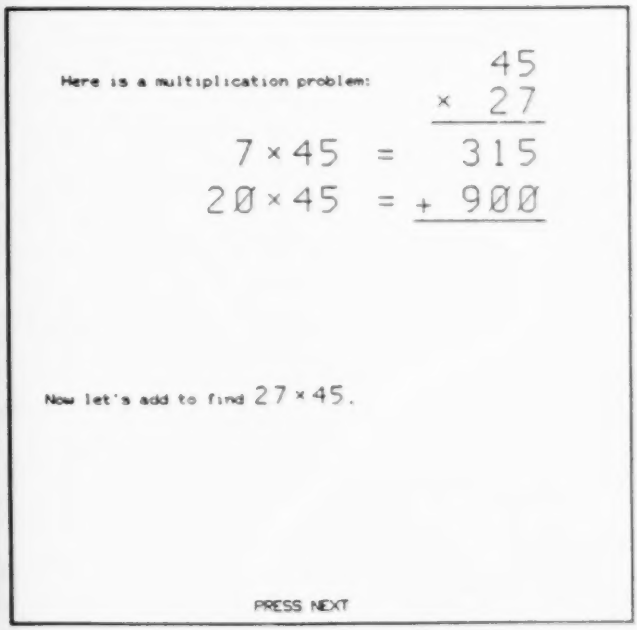



**Figure 3**

**Figure 4**

**Lesson** **Speedway! [Add, Subtract, Multiply, Divide]**

**File** *speedway*

**Author** Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To enhance speed and accuracy in answering math facts.

**Description** This lesson has been described in Unit 5, pages 36 through 38.

# Unit 7

## Dividing by Ones Numbers

**Objective** To divide by a one-digit number, sometimes leaving a remainder and sometimes not.

**Description** This unit serves as an introduction to division. Students are taught the concept and process of division. They learn to estimate dividends. Also, they learn and practice division facts.

**Lessons**

- Facts Practice [Division]
  {Division Is ...}
  {Gimme!}
  {Dividing by Trying Numbers to See If They Fit}
  Introduction to Long Division: Part 1
  {Speedway! [Add, Subtract, Multiply, Divide]}

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | Learning to Divide |
| 3A, | 3B, | 3C | Dividing Tens Numbers |
| 5A, | 5B, | 5C | Dividing Hundreds Numbers |
| 7A, | 7B, | 7C | Zero in the Middle & Dividing into Thousands Numbers |
| 9A, | 9B, | 9C | When It Looks As If You Cannot Divide |
| 11A, | 11B, | 11C | Numbers That Cannot Be Divided Evenly |
| 13A, | 13B, | 13C | Carrying the Remainder with Tens Place Numbers |
| 15A, | 15B, | 15C | Carrying the Remainder with Hundreds Place Numbers |

{ } Lesson will not be assigned unless the instructor chooses to do so. (See Preface for explanation.)

**Lesson** **Facts Practice [Division]**

(*SYS 4 catalog name*: **Division Math Facts Drill**)

**File** *facts (SYS 4 file name: factspcp.10)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Helen Leung
Martin A. Siegel
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
and Courseware & Curriculum Applications (CCA) Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To memorize the division facts, with response time of less than three seconds.

**Description** Students practice division facts in vertical form. Speed, as well as accuracy, is required for successful completion (see Figure 1).

**Interaction**

- Students work with sets of 10 division facts, so that groups of facts are practiced and mastered.
- Both forms of commutative facts are given.
- If a fact is answered correctly twice in a row, it is displayed as a mastered fact on the screen.
- If a fact is mastered without errors, the chart is marked with an asterisk.
- A second asterisk is given if the final iteration is answered in less than three seconds.
- If a fact is missed, feedback is given with the correct answer. Then, that fact is returned to the item queue and will recur at increasing intervals. This helps ensure mastery of each fact.
- A display of the facts students have mastered is given after they complete each set of problems (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 25 minutes (self-paced)

**Associated Lessons** *Facts Practice [Addition]*
*Facts Practice [Subtraction]*
*Facts Practice [Multiplication]*
*Speedway! [Add, Subtract, Multiply, Divide]*

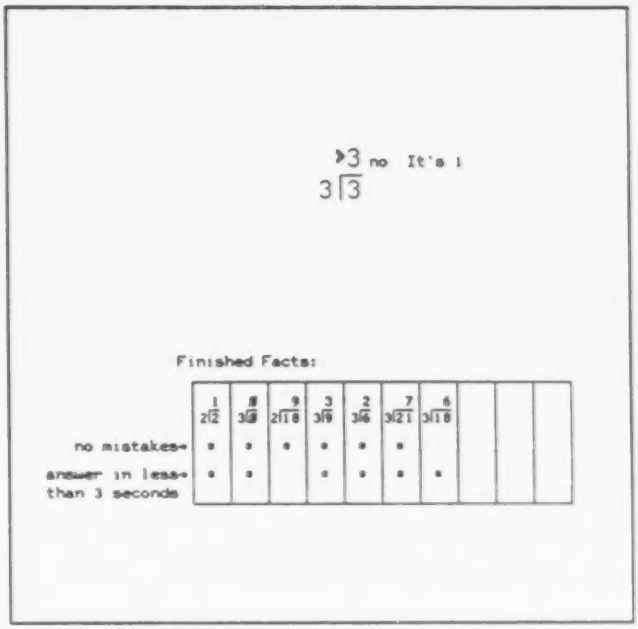



**Figure 1**

denise natour's Division Facts
You have completed 2 sessions of division facts practice.
You have mastered 1's, and 2's.
Press NEXT to leave
or LAB to do another set of problems.

**Figure 2**

**Lesson** **Division Is ...**

(*SYS 4 catalog name*: **Meaning of Division**)

**File** *divmeans*

**Authors** Charles S. Weaver
Bonnie A. Seiler
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To visualize the division process.
- To write division number sentences.
- To work division problems with remainders.

**Description** This six-part lesson uses pictures to explain that division is a process of dividing a number into smaller but equal parts. Sometimes there are left-overs, which are left as remainders. The lesson bridges the gap between division problems represented in pictures and the division algorithm.

**Interaction**

- Pictures and story lines help students visualize the division process (see Figures 1 and 2).
- Students can press HELP for assistance at some points in the program (see Figure 3).
- If students miss an answer three times in a row, they are given the correct answer.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (open-ended, minimum of 10 minutes required)

**Associated Lesson** *Gimme!*

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

**Figure 1**

1 cake
Here are 16 cakes.
We want to divide these cakes fairly among 3 ants.
How many whole cakes will each ant get?
5 cakes apiece, and 1 cake left over.
Press -NEXT- to see a division sentence about dividing cakes.

**Figure 2**

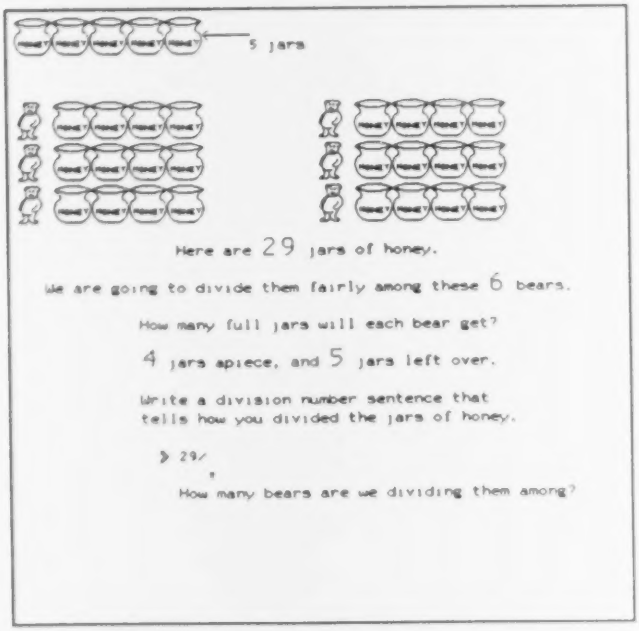

5 jars
Here are 29 jars of honey.
We are going to divide them fairly among these 6 bears.
How many full jars will each bear get?
4 jars apiece, and 5 jars left over.
Write a division number sentence that tells how you divided the jars of honey.
> 29/
How many bears are we dividing them among?

**Figure 3**

**Lesson** **Gimme!**

**File** *gimme*

**Authors** Charles S. Weaver
Bonnie A. Seiler
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To understand the concept of division as sharing equally among groups.

**Description** In this lesson, students are given a large number of boxes (they choose the contents) and are asked to share them fairly among a group of friends. The purpose of the lesson is to help students visualize the division process.

Students are asked to give one box to each friend, one at a time; then, they calculate the number of boxes distributed and the number remaining. Students do several problems; some have remainders, some do not.

**Interaction**

- Students follow prompts on the screen to distribute boxes and to figure how many are left after each distribution (see Figure 1).
- If students answer a question incorrectly three times in row, they are given the right answer.
- The program allows students to make wrong choices. The incorrectness is then explained through visual representation of the results (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (open-ended, minimum of 10 minutes required)

**Associated Lesson** *Division Is ...*

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

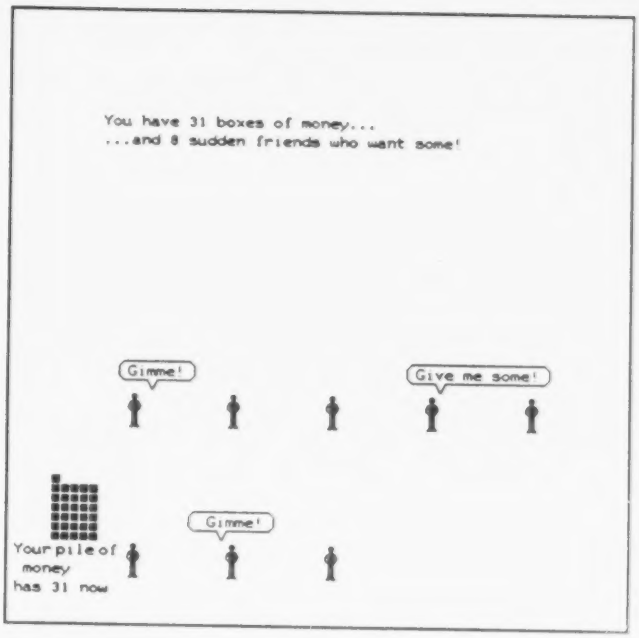



**Figure 1**

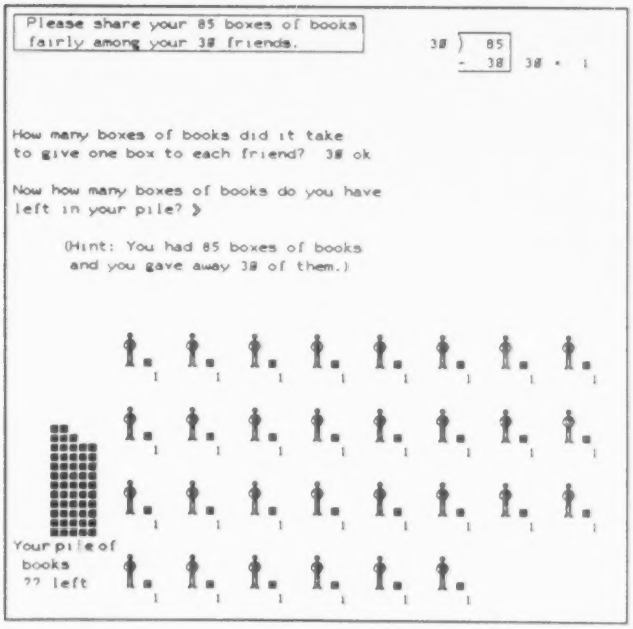

Please share your 85 boxes of books fairly among your 38 friends.
38 ) 85
- 38
38 • 1
How many boxes of books did it take to give one box to each friend? 38 ok
Now how many boxes of books do you have left in your pile? )
(Hint: You had 85 boxes of books and you gave away 38 of them.)
Your pile of books ?? left

**Figure 2**

**Lesson** ## Dividing by Trying Numbers to See If They Fit

(*SYS 4 catalog name*: **Division by Trying a Number**)

**File** *trynum*

**Authors** Charles S. Weaver
Bonnie A. Seiler
Sue Campanini

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To work division problems by guessing quotients and multiplying to see if the quotient fits.

**Description** In this lesson, students choose or are given division problems. They check trial quotients by multiplying to see if their chosen divisors fit into the dividend. A list of these products is kept on the screen. Students decide when they are ready to be judged on their answers.

Students may do as many new problems as they wish before leaving the lesson.

**Interaction**

- In the first part of the lesson, the computer does the multiplication. In the second part, students multiply for themselves.
- Students work with a problem until they wish to check their answer. If it is not correct, they are guided to the right answer (see Figures 1 through 3).
- When the program asks students to write their own division problem, they can choose to let the computer write one for them. They can then choose an easy, medium, or hard problem.
- Scratch paper may be needed when students solve their own problems.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

**Figure 1**

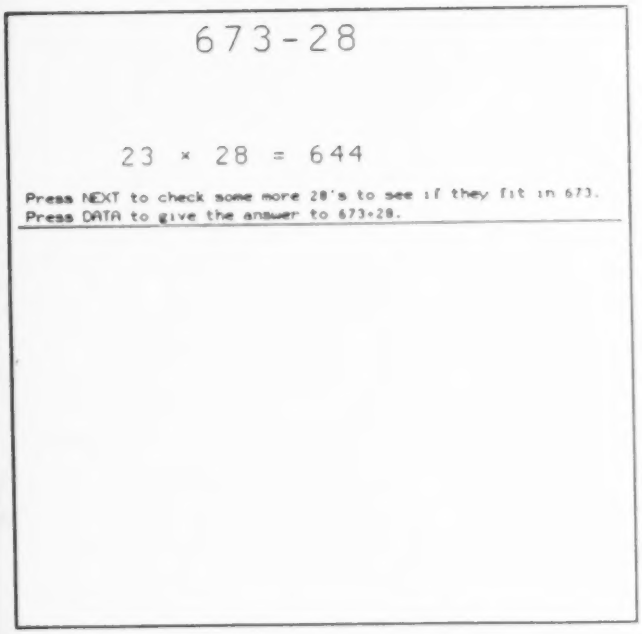



**Figure 2**

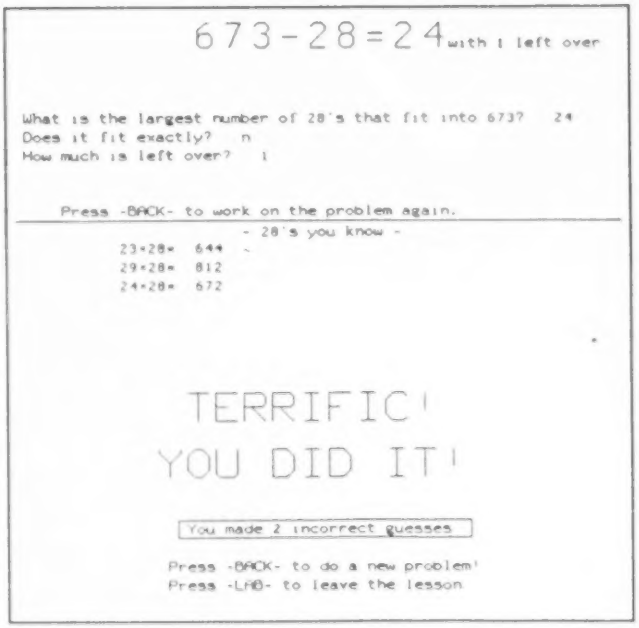

673-28=24 with 1 left over
What is the largest number of 28's that fit into 673? 24
Does it fit exactly? n
How much is left over? 1
Press -BACK- to work on the problem again.
- 28's you know -
23×28= 644
29×28= 812
24×28= 672
TERRIFIC!
YOU DID IT!
You made 2 incorrect guesses
Press -BACK- to do a new problem!
Press -LAB- to leave the lesson

**Figure 3**

**Lesson** ## Introduction to Long Division: Part 1

(*SYS 4 catalog name*: **Long Division 1**)

**File** *div1*

**Authors** Charles S. Weaver
Bonnie A. Seiler
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To show that division is a process of dividing a larger number into smaller but equal parts, with or without a remainder.
- To divide, using pictorial representations.

**Description** This lesson introduces long division in terms of visual models of equally shared objects (see Figure 1). Students are taught to use the standard long division process in *Introduction to Long Division: Part 2* and *Introduction to Long Division: Part 3* in Unit 8.

**Interaction**

- Students are given many opportunities to guess at division facts. If they are wrong, the program shows them why the error is wrong instead of merely giving them the correct answer (see Figure 2).
- After three incorrect answers to any one question, the program tells students the correct answer.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (open-ended, minimum of 15 minutes required)

**Associated Lessons** *Introduction to Long Division: Part 2*
*Introduction to Long Division: Part 3*

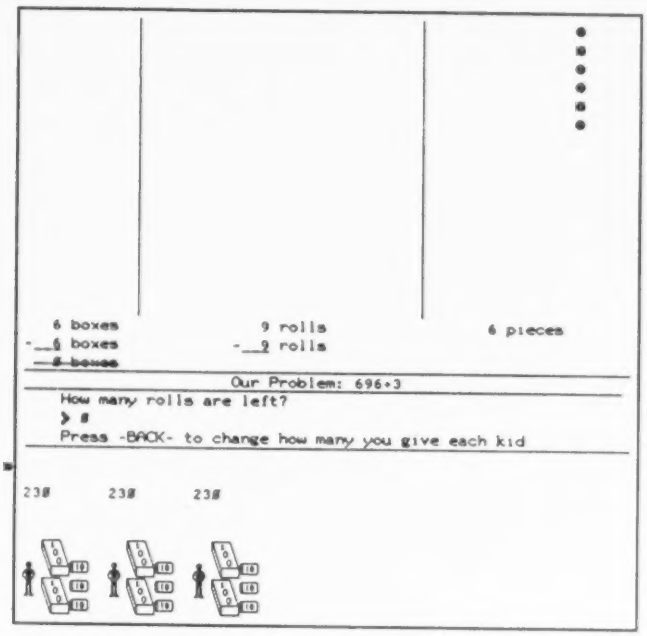



**Figure 1**

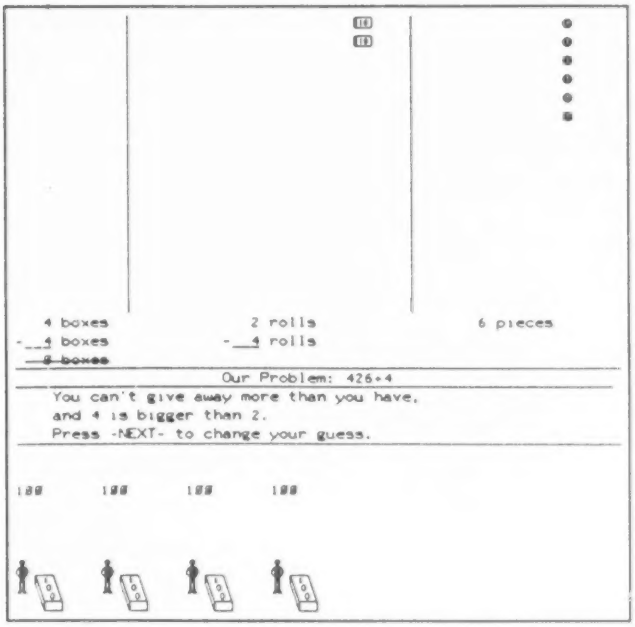



**Figure 2**

**Lesson** **Speedway! [Add, Subtract, Multiply, Divide]**

**File** *speedway*

**Author** Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To enhance speed and accuracy in answering math facts.

**Description** This lesson has been described in Unit 5, pages 36 through 38.

**Special Note** This lesson will not be assigned unless the instructor chooses to do so. However, it will be automatically assigned in Unit 8 (if students need work in Unit 8).

# Unit 8

## Dividing by Tens or Hundreds Numbers

**Objective** To divide by two-digit and three-digit numbers, sometimes leaving remainders and sometimes not.

**Description** Students work division problems in which the divisor has more than one digit. They also continue to learn and practice division facts.

**Lessons**
- Facts Practice [Division]
  Introduction to Long Division: Part 2
  Introduction to Long Division: Part 3
  Speedway! [Add, Subtract, Multiply, Divide]

**Offline Materials**
- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | Dividing by Tens Numbers, No Remainders |
| 3A, | 3B, | 3C | Dividing by Tens Numbers, With Remainders |
| 5A, | 5B, | 5C | Dividing Twice |
| 7A, | 7B, | 7C | Dividing Three or Four Times |
| 9A, | 9B, | 9C | Dividing by Hundreds Numbers, Multiple Times |

**Lesson** **Facts Practice [Division]**

(*SYS 4 catalog name*: **Division Math Facts Drill**)

**File** *facts (SYS 4 file name: factspcp.10)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Helen Leung
Martin A. Siegel
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
and Courseware & Curriculum Applications (CCA) Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To memorize the division facts, with response time of less than three seconds.

**Description** This lesson has been described in Unit 7, pages 51 and 52.

**Note**: If this lesson has been successfully completed in Unit 7, it will not be assigned in Unit 8. If students have tested out of Unit 7, and thus have not completed this drill, then it will be assigned.

**Lesson** ## Introduction to Long Division: Part 2

(*SYS 4 catalog name*: **Long Division 2**)

**File** *div2*

**Authors** Charles S. Weaver
Bonnie Anderson Seiler
J. Michael Felty

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To do division problems using the standard division algorithm.

**Description** Students correlate a model of division (in the form of a story problem) with the long division algorithm. Only single-digit divisors are used.

**Interaction**

- Students are led to an understanding of the standard division algorithm by associating tangible objects grouped by 10s and 100s with place value (see Figures 1 and 2).
- Students practice the standard division algorithm to ensure that they understand the concept (see Figure 3).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (open-ended, minimum of 15 minutes required)

**Associated Lessons** *Introduction to Long Division: Part 1*
*Introduction to Long Division: Part 3*

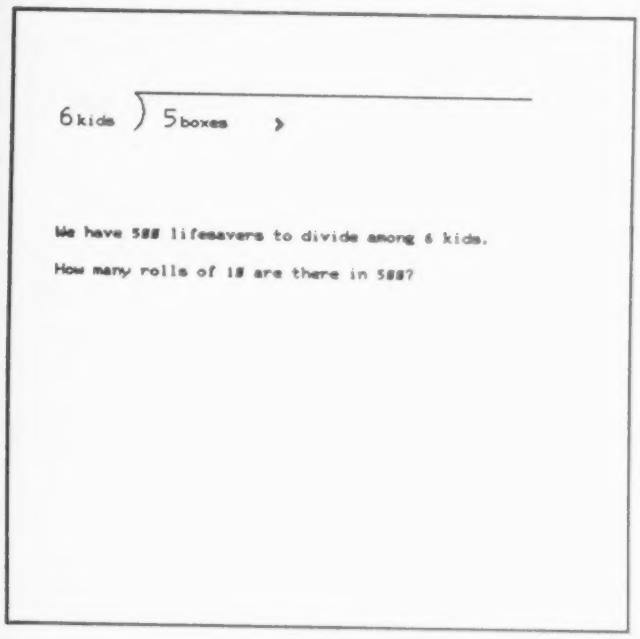



Figure 1

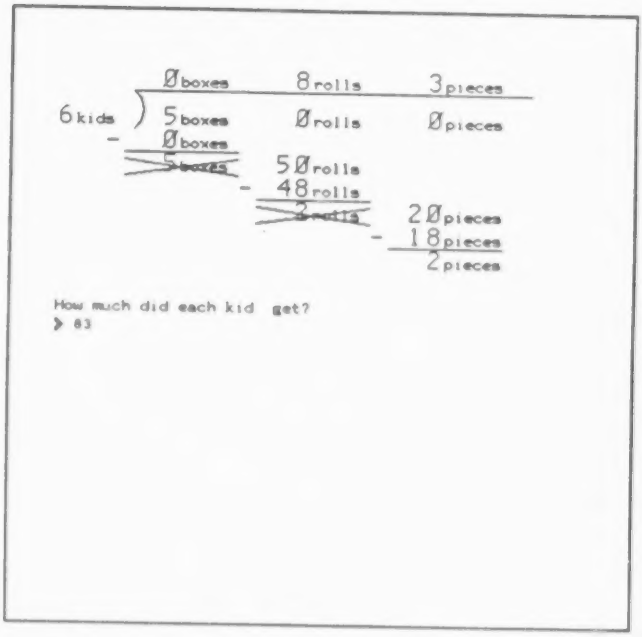

0 boxes 8 rolls 3 pieces
6 kids ) 5 boxes 0 rolls 0 pieces
- 0 boxes
5 0 rolls
- 48 rolls
2 0 pieces
- 18 pieces
2 pieces
How much did each kid get?
> 83

Figure 2

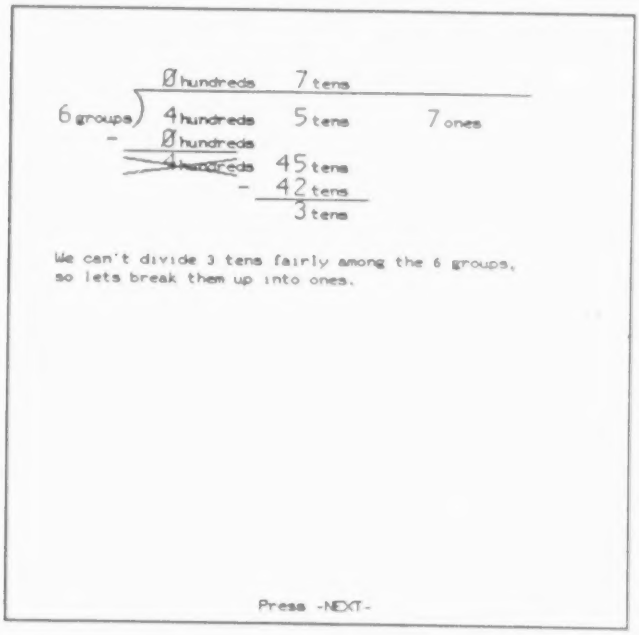

0 hundreds 7 tens
6 groups ) 4 hundreds 5 tens 7 ones
- 0 hundreds
45 tens
- 42 tens
3 tens
We can't divide 3 tens fairly among the 6 groups,
so lets break them up into ones.
Press -NEXT-

Figure 3

**Lesson:** ## Introduction to Long Division: Part 3

(*SYS 4 catalog name*: **Long Division 3**)

**File** *divalg*

**Authors** Charles S. Weaver
Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To divide whole numbers with up to five-digit divisors and dividends, with and without remainders.

**Description** Students practice division problems of their choice using the standard division algorithm.

**Interaction**

- Students write problems of their own or press HELP to have the computer provide problems.
- Students are prompted with an arrow to guide them in the division process (see Figure 1).
- If students make errors, they are given hints to help them answer correctly (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 25 minutes (open-ended, minimum of 20 minutes required)

**Associated Lessons** *Introduction to Long Division: Part 1*
*Introduction to Long Division: Part 2*

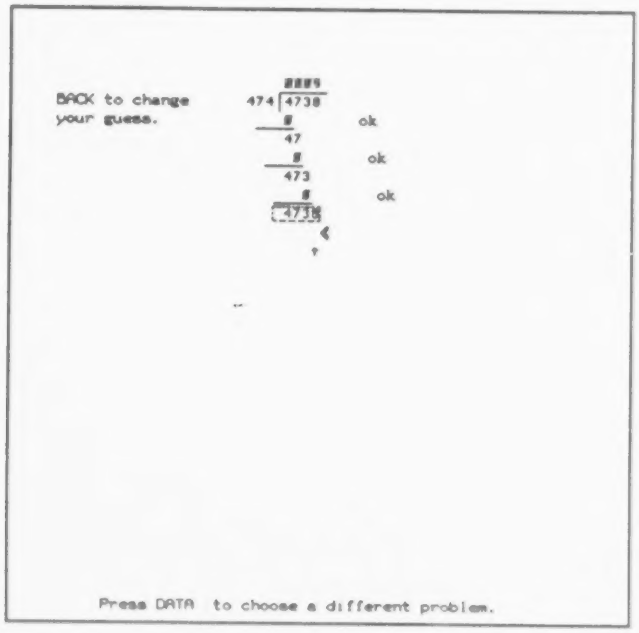



Figure 1

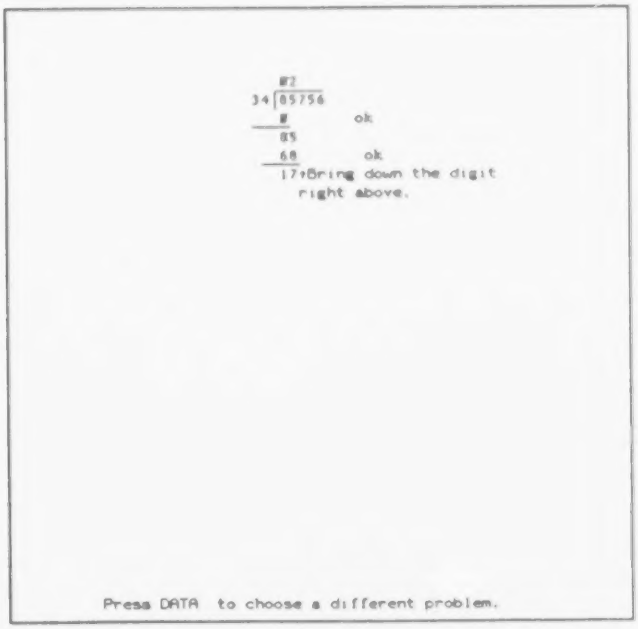



Figure 2

**Lesson** **Speedway! [Add, Subtract, Multiply, Divide]**

**File** *speedway*

**Author** Bonnie Anderson Seiler

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To enhance speed and accuracy in answering math facts.

**Description** This lesson has been described in Unit 5, pages 36 through 38.



86

# Unit 9

## Consolidation #1

**Objective** To review the whole number skills taught in Units 1 through 8.

**Description** The instructor should review students' tests and assign review materials corresponding to the type of items missed. Suggested lessons for further study are listed below. All have been described in previous units except for *Pinball Number Facts Drill*, which is described here.

There are no off-line worksheets specific to this unit. Instructors should make appropriate assignments from the worksheets in previous units, or assign work from other appropriate materials.

NOTE: NONE OF THESE LESSONS WILL BE ASSIGNED UNLESS THE INSTRUCTOR SPECIFICALLY CHOOSES TO DO SO.

**Lessons**

- There is one new lesson: {Pinball Number Facts Drill}
- The lessons for students to review from previous units include:

| Title of Lesson | Unit Number | File Name | Page Number |
|---|---|---|---|
| {Checkup on Addition} | 2 | *addchk* | 10 |
| {Subtraction with Sticks} | 4 | *sticksv* | 20 |
| {Multiplying by a One-Digit Number} | 5 | *multa1* | 32 |
| {Speedway!} | 5 | *speedway* | 36 |
| {Multiplying by Numbers Ending in Zero} | 6 | *multa2* | 45 |
| {Multiplying by Two-Digit Numbers} | 6 | *multalg* | 47 |
| {Division Is ...} | 7 | *divmeans* | 53 |
| {Introduction to Long Division: Part 1} | 7 | *div1* | 59 |
| {Introduction to Long Division: Part 2} | 8 | *div2* | 64 |
| {Introduction to Long Division: Part 3} | 8 | *divalg* | 66 |

{ } Lesson will not be assigned unless the instructor chooses to do so. (See Preface for explanation.)

**Lesson** **Pinball Number Facts Drill**

**File** *pinw*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice addition, subtraction, multiplication, and division facts. Students are rewarded for speed and accuracy.

**Description** Students practice basic number facts as a ball bounces from problem to problem as in a pinball machine (see Figure 1). Points are earned by answering each problem correctly within a time limit, which adjusts to student performance.

There are five parts in this math game. Students answer questions within a part, until they miss four answers. The rounds presented in order are: addition subtraction, multiplication, division, and mixed facts.

**Interaction**

- A simulated pinball indicates the math fact to be answered. Students are allowed up to three tries for an answer (see Figure 2).
- Answering quickly shortens the time limit for each problem.
- Depending upon the location of each arithmetic problem on the page, students earn 20, 30, 40, or 50 points for each correct answer.
- A list of students with the 20 highest scores is available (see Figure 3).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (open-ended, minimum of 10 minutes required)

**Associated Lessons**
*Facts Practice [Addition]*
*Facts Practice [Subtraction]*
*Facts Practice [Multiplication]*
*Facts Practice [Division]*
*Speedway!*
*Decimal Pinball*

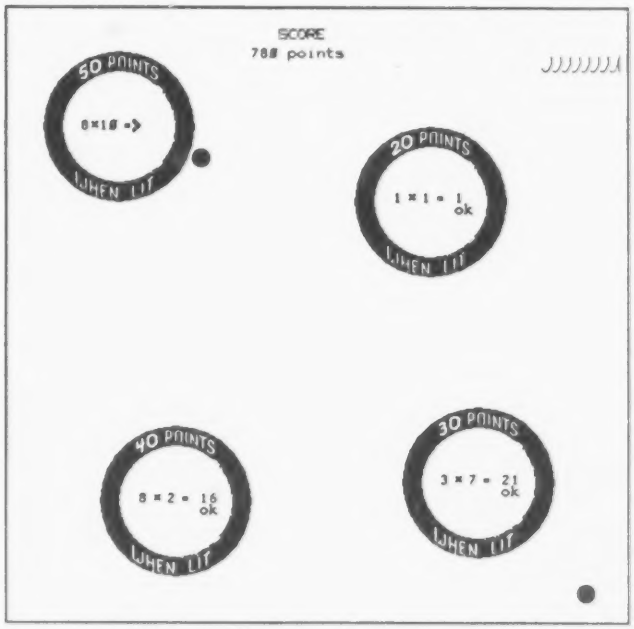



**Figure 1**

**Figure 2**

PINBALL HALL OF FAME

| | Name | Group | Score | Date |
|---|---|---|---|---|
| 1 | student | dpl | 5150 | January 25, 1990 |
| 2 | kfoss | mainei | 4930 | December 22, 1989 |
| 3 | keppel e | mainep | 4780 | November 17, 1989 |
| 4 | linnehan r | mainei | 4690 | January 14, 1990 |
| 5 | doyle | smvti | 4610 | November 8, 1989 |
| 6 | visitor | brewerhs | 4440 | December 11, 1989 |
| 7 | hunt r | mainei | 4350 | February 13, 1990 |
| 8 | visitor | hancock | 4130 | January 9, 1990 |
| 9 | visitor | nhs | 4100 | November 10, 1989 |
| 10 | miller c | sacchem1 | 3980 | December 8, 1989 |
| 11 | visitor | visitor | 3810 | November 27, 1989 |
| 12 | tim peterson | nha | 3740 | February 7, 1990 |
| 13 | hesseltine j | mainei | 3700 | November 22, 1989 |
| 14 | visitor | mainenet | 3670 | November 15, 1989 |
| 15 | steven c | freone | 3590 | January 8, 1990 |
| 16 | joe higuera | tmd | 3560 | February 2, 1990 |
| 17 | balmain sean | tmd | 3470 | September 26, 1989 |
| 18 | farwell | euclid | 3450 | November 3, 1989 |
| 19 | ayer | euclid | 3170 | November 3, 1989 |
| 20 | alpert t | mainei | 3150 | January 27, 1990 |

This Hall of Fame was started on August 23, 1989.

**Figure 3**

**Pinball Number Facts Drill**

# Unit 10

# What Is a Fraction?

**Objectives**

- To change improper fractions to mixed numbers, and vice versa.
- To reduce fractions to lowest terms.

**Description**

This unit is an introduction to fractions and mixed numbers. Students are taught what fractions, mixed numbers, and whole numbers are and how they differ from one another. Students are taught to recognize and find equivalent fractions and mixed numbers.

The lessons in this unit are divided into three groupings. The first group covers the basic concept: "What is a fraction?" The second group covers basic mixed numbers and the concept of multiple names for the same number (equivalent fractions sets). The third group covers simplest terms and gives more practice in finding multiple names for the same number.

Lessons assigned from the first group will appear in an increasing index with review allowed (as lessons normally do in this curriculum). When all lessons assigned from the first group are completed, then, lessons assigned from the second group will begin to appear. At this point, the lessons from the first group will no longer be available.

Students then work through the second set of lessons in the usual way until they are completed. Then, the third and final set begins to appear. When the lessons from the third group are completed, all lesson work for the unit is done, and students move on to the posttest.

**Lessons**

- Cut and Paint
  Fraction Notation
  Lights
  Lights: Equivalent Fractions 1
  Checkup: Cut and Paint
  Checkup: Lights

- Pizza: Mixed Numbers Introduction
  Try These: Fractions and Mixed Numbers
  (Try These: Equivalent Mixed Numbers)
  Vocabulary: Whole Number, Fraction, Mixed Number
  Sort Numbers by Form
  Checkup: Mixed Numbers with Pizza
  (Checkup: Equivalent Mixed Numbers)

- Lights: Equivalent Fractions 2
  Boxes: Name Equivalent Fractions
  Sort Equivalent Fractions
  Sort Equivalent Mixed Numbers
  {Try These: Equivalent Fractions}
  Introduction to Simplest Terms
  Pizza: Common Denominator
  Checkup: Sort Equivalent Fractions
  {Checkup: Try These Equivalent Fractions}
  {Checkup: Simplest Terms}

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | What Is a Fraction? |
| 3A, | 3B, | 3C | Changing Improper Fractions to Mixed Numbers |
| 5A, | 5B, | 5C | Changing Mixed Numbers to Improper Fractions |
| 7A, | 7B, | 7C | Reducing Fractions |
| 9A, | 9B, | 9C | Review of Fractions Concepts |

---

{ } Lesson will not be assigned unless the instructor chooses to do so. (See Preface for explanation.)

**Lesson** ## Cut and Paint

**File** *rec (SYS 4 file name: rec.1)*

**Authors** Sharon Dugdale
David Kibbey
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To represent a fraction as a portion of a rectangle.

**Description** Students construct fractions by *painting* parts of boxes. Items range from *one-half* to *five-fifths* to *seven-twelfths* to *zero-sixths*.

**Interaction**

- Students create a grid structure and then paint and remove paint to create the appropriate fraction.
- If students make an error, they are given appropriate feedback and asked to try again (see Figure 1).
- Correct answers bring harder problems; missed items bring easier problems.
- Items are generated until students achieve mastery.
- Students have many options for creativity as they may use different grids to create the same fraction (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons**
*Checkup: Cut and Paint*
*Boxes: Name Equivalent Fractions*
*Paint Addition: The Computer Paints*
*Paint Addition: You Paint*
*Checkup: Paint Addition*

**Special Note** Touch capability required

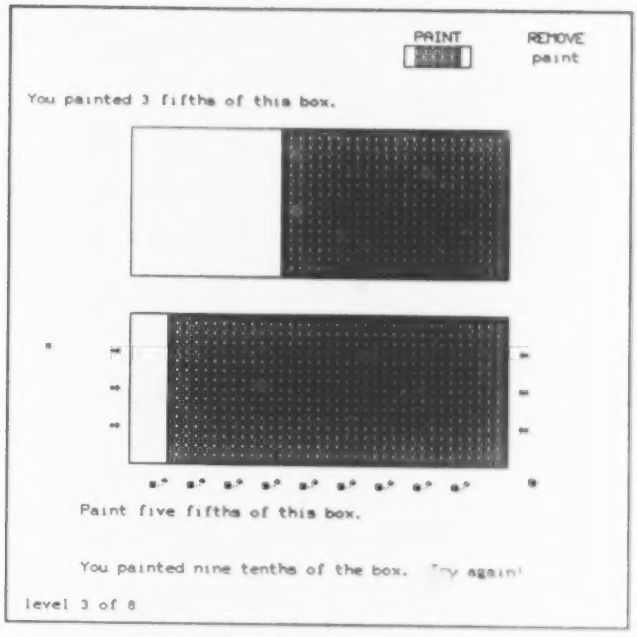

PAINT
REMOVE paint
You painted 3 fifths of this box.
Paint five fifths of this box.
You painted nine tenths of the box. Try again!
level 3 of 8

Figure 1

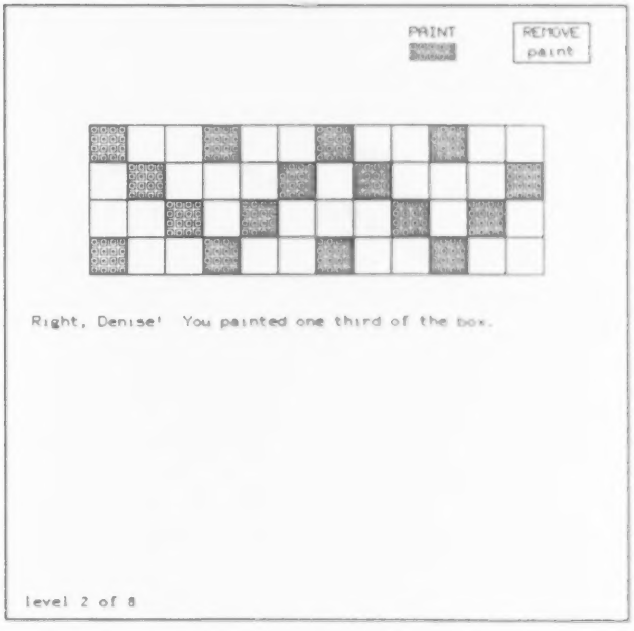



Figure 2

**Lesson** **Fraction Notation**

**File** *fnotation (SYS 4 file name: fnotation.1)*

**Authors** Sharon Dugdale
David Kibbey
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To write fractions in x/y notation, given the equivalent fraction in words.

**Description** Students practice writing fractions using the notation taught.

**Interaction**

- Students are helped to find the slash key (/) on the keyboard, if they are having problems with the notation (see Figure 1).
- If students make a mistake, the correct answer is given, and they must retype the answer, reinforcing the correct response (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 2 minutes (self-paced)

Figure 1

Figure 2

**Lesson** **Lights**

(*SYS 4 catalog name*: **Lights: Basic Fractions**)

**File** *lites (SYS 4 file name: lites.1)*

**Authors** Sharon Dugdale
David Kibbey
Susan Frasier Lamprinos
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To work with pictorial representations of fractions.

**Description** First, students see a set of light bulbs and are asked to turn a fraction of them on.

Later, a set of light bulbs is presented with some of them already turned on. Students are asked to tell what fraction of the light bulbs are turned on.

**Interaction**

- Students touch the appropriate number of light bulbs to construct a picture of the target fraction (see Figure 1).
- Given a picture, students tell what fraction of light bulbs are turned on (see Figure 2).
- If students make a mistake, they are led to the correct answer (see Figure 3).
- Lesson difficulty adjusts to student performance.
- Items are generated until students achieve mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Checkup: Lights*
*Lights: Equivalent Fractions 1*
*Lights: Equivalent Fractions 2*

**Special Note** Touch capability required

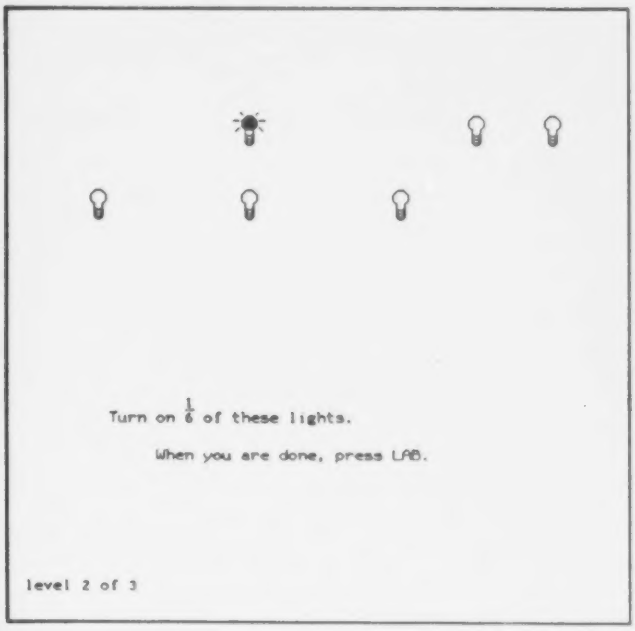



Figure 1

Figure 2

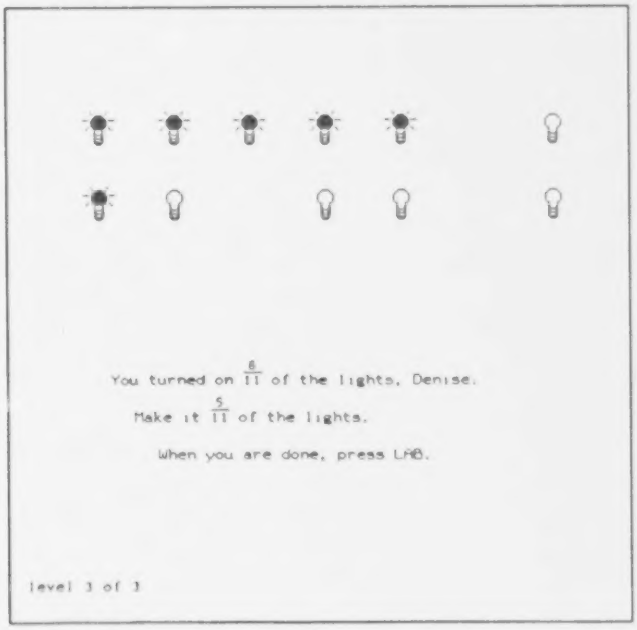



Figure 3

**Lesson** ## Lights: Equivalent Fractions 1

**File** *liteseq (SYS 4 file name: liteseq.1)*

**Authors** Sharon Dugdale
David Kibbey
Susan Frasier Lamprinos
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To learn the concept of equivalent fractions.

**Description** This lesson uses the same format as the earlier lesson, *Lights*. This lesson, however, adds the dimension of equivalent fractions. Students learn to understand equivalent fractions by observing, for example, that five out of ten light bulbs are the same as 1/2 (see Figure 1).

**Interaction**

- Students touch the appropriate number of light bulbs to construct a picture of the target fraction.
- Students tell how many bulbs they turned on to achieve the equivalent fraction requested. Later, they must also tell how many lights in the set (i.e., give both numerator and denominator).
- Light bulbs appear in arrays, making visualization of the equivalencies easier.
- If students make a mistake in determining the equivalent fraction, they are guided toward the correct answer (see Figure 2).
- Lesson difficulty adjusts to student performance.
- Items are generated until students achieve mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons** *Lights*
*Checkup: Lights*
*Lights: Equivalent Fractions 2*

**Special Note** Touch capability required

Turn on 1/2 of the lights.
Good, Denise.
You turned on 5/10 of the lights.
Yes, 1/2 of the lights is the same as 5/10 of the lights.
GOOD! Press NEXT.
level 3 of 10

**Figure 1**

**Figure 2**

**Lesson** ## Checkup: Cut and Paint

**File** *recchk (SYS 4 file name: recchk.1)*

**Authors** Sharon Dugdale
David Kibbey
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To represent a fraction as a portion of a rectangle.

**Description** As in the earlier lesson, *Cut and Paint*, students construct fractions by *painting* parts of boxes (see Figure 1). This lesson is a review checkup. Only the more difficult items appear in this version (*seven-twelfths*, *zero-sixths*, etc.).

**Interaction**

- Students create a grid structure and then paint or remove paint to create the appropriate fraction.
- If students make an error, they are given appropriate feedback and asked to try again (see Figure 2).
- Students have many options for creativity as they may use different grids (see Figure 3).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (self-paced)

**Associated Lessons**
*Cut and Paint*
*Boxes: Name Equivalent Fractions*
*Paint Addition: The Computer Paints*
*Paint Addition: You Paint*
*Checkup: Paint Addition*

**Special Note** Touch capability required

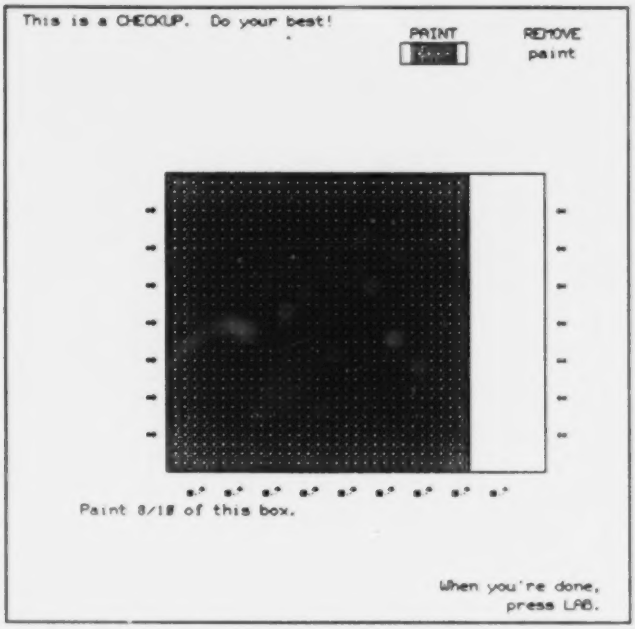



**Figure 1**

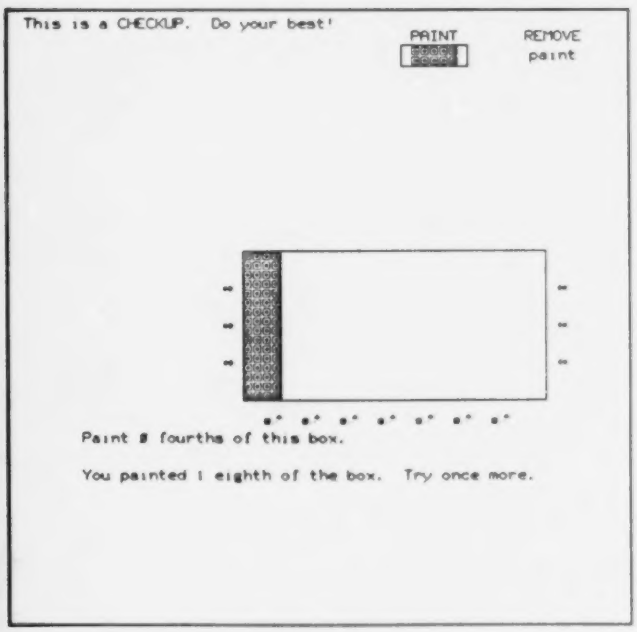



**Figure 2**

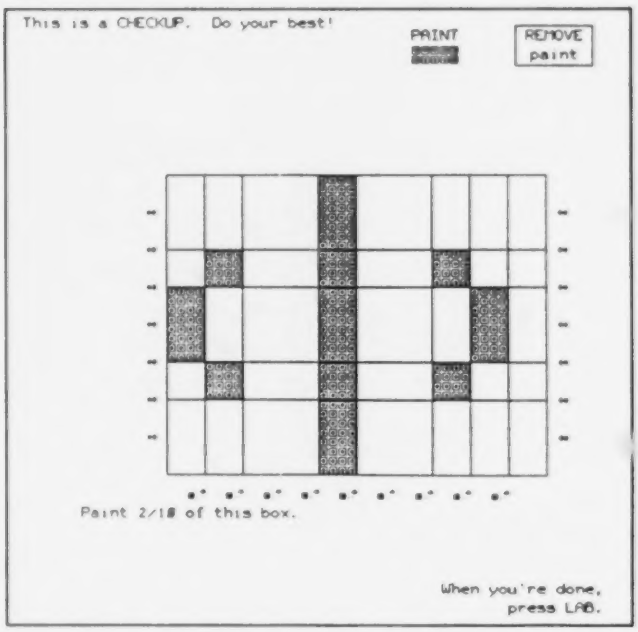



**Figure 3**

**Lesson** ## Checkup: Lights

**File** *lites (SYS 4 file name: lites.2)*

**Authors** Sharon Dugdale
David Kibbey
Susan Frasier Lamprinos
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To work with pictorial representations of fractions.

**Description** Students are tested on concepts presented in *Lights*. First, students are given a set of light bulbs and asked to turn a fraction of them on.

Later, a set of light bulbs is presented with some of them turned on. Students are asked to tell what fraction of the light bulbs are turned on.

**Interaction**

- Students touch the appropriate number of light bulbs to construct a representation of the target fraction (see Figure 1).
- Given a picture, students tell what fraction of light bulbs are turned on (see Figure 2).
- If students make a mistake, they are led to the correct answer (see Figure 3).
- Items are similar to those at the higher levels of *Lights*.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Lights*
*Lights: Equivalent Fractions 1*
*Lights: Equivalent Fractions 2*

**Special Note** Touch capability required

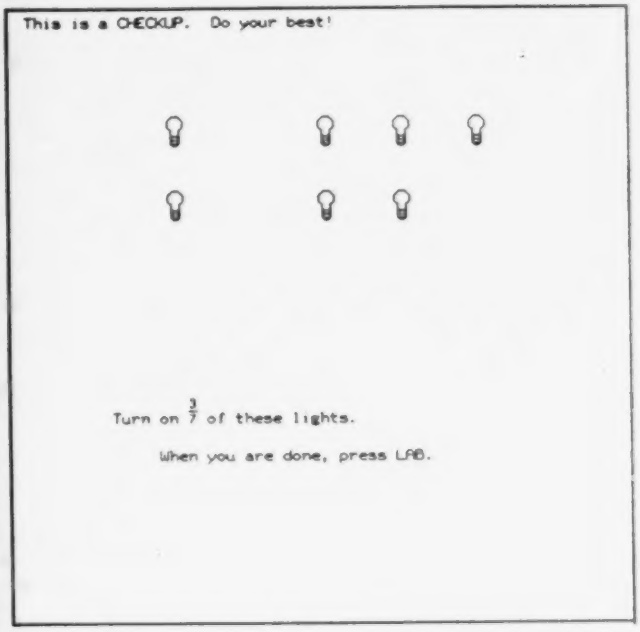



Figure 1

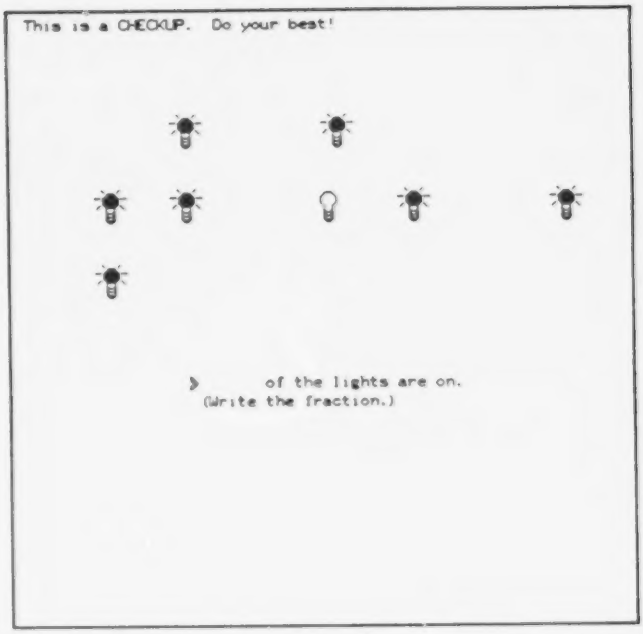



Figure 2

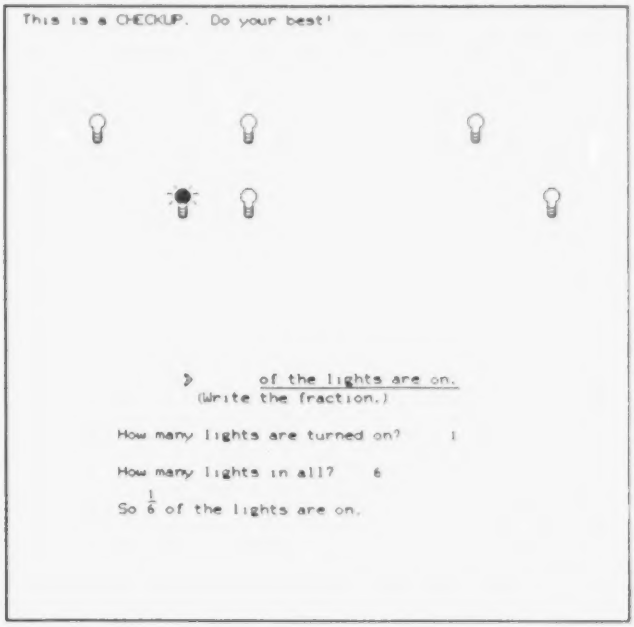

This is a CHECKUP. Do your best!
of the lights are on.
(Write the fraction.)
How many lights are turned on? 1
How many lights in all? 6
So 1/6 of the lights are on.

Figure 3

**Lesson** **Pizza: Mixed Numbers Introduction**

**File** *pizza4 (SYS 4 file name: pizza4.1)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung
Paul Cohen
Susan Frasier Lamprinos

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To convert an improper fraction to a mixed number.
- To convert a mixed number to an improper fraction.

**Description** Students learn to recognize that a mixed number and a fraction can name the same number. Students must divide whole and partial pizzas equally between two people (see Figure 1). Students practice converting mixed numbers to fractions and vice versa. Initially, pictures are used to help students visualize the problem.

**Interaction**

- If errors are made, feedback guides students to the right answer (see Figure 2).
- Students progress to problems without on-screen dialog (see Figure 3) and then to problems without pictures.
- Current student level is displayed on the screen. This level adjusts up or down based on student performance.
- Items are generated until students reach mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons** *Checkup: Mixed Numbers with Pizza*
*Pizza: Common Denominator*

**Special Note** Touch capability required

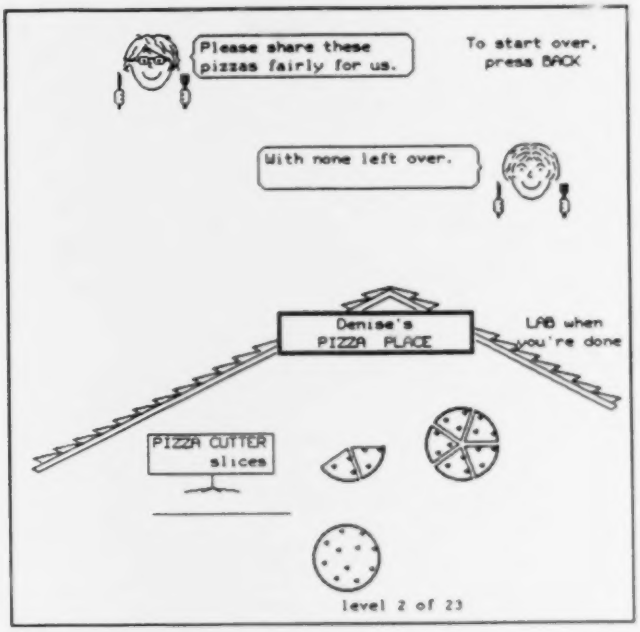

Please share these pizzas fairly for us.
To start over, press BACK
With none left over.
Denise's PIZZA PLACE
LAB when you're done
PIZZA CUTTER slices
level 2 of 23

**Figure 1**

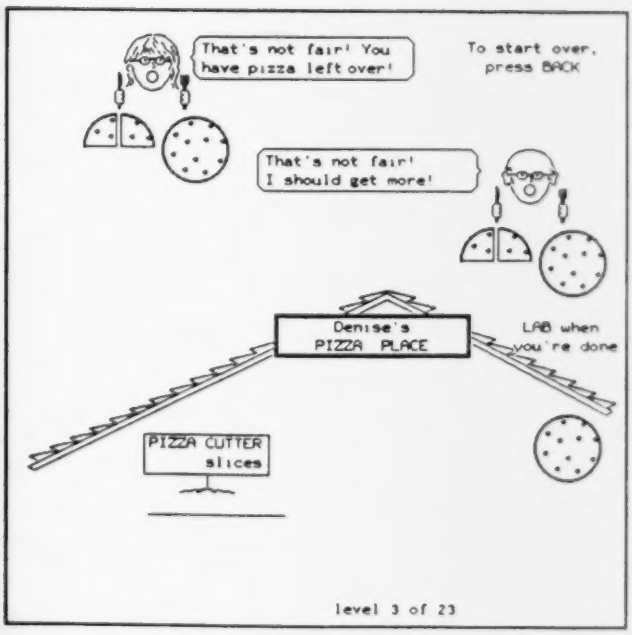

That's not fair! You have pizza left over!
To start over, press BACK
That's not fair! I should get more!
Denise's PIZZA PLACE
LAB when you're done
PIZZA CUTTER slices
level 3 of 23

**Figure 2**

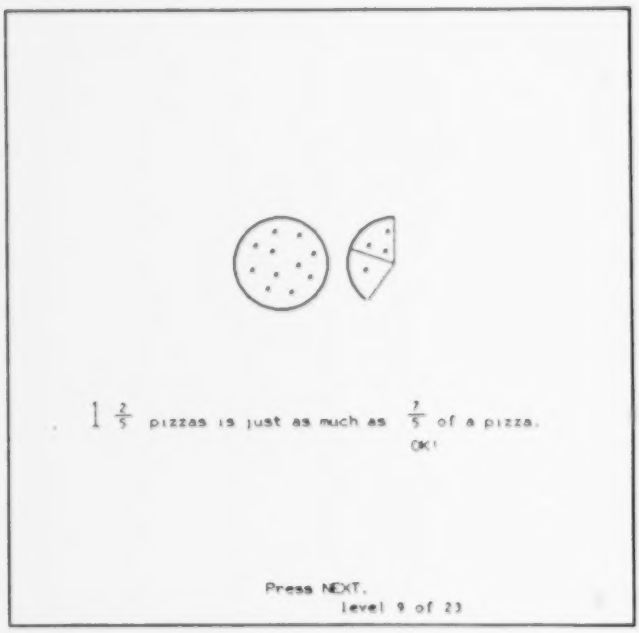



**Figure 3**

**Lesson** **Try These: Fractions and Mixed Numbers**

**File** *tryeq (SYS 4 file name: tryeq.3)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To convert improper fractions to mixed numbers.
- To convert mixed numbers to improper fractions.

**Description** Students are presented with four problems per page and are prompted to fill in the blanks for missing numerators, denominators, and whole numbers (see Figure 1).

**Interaction**

- If students answer incorrectly twice, the computer gives the correct answer (see Figure 2).
- Items are generated until students reach mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons**
*Try These: Equivalent Mixed Numbers*
*Checkup: Equivalent Mixed Numbers*
*Try These: Equivalent Fractions*
*Checkup: Try These: Equivalent Fractions*



106

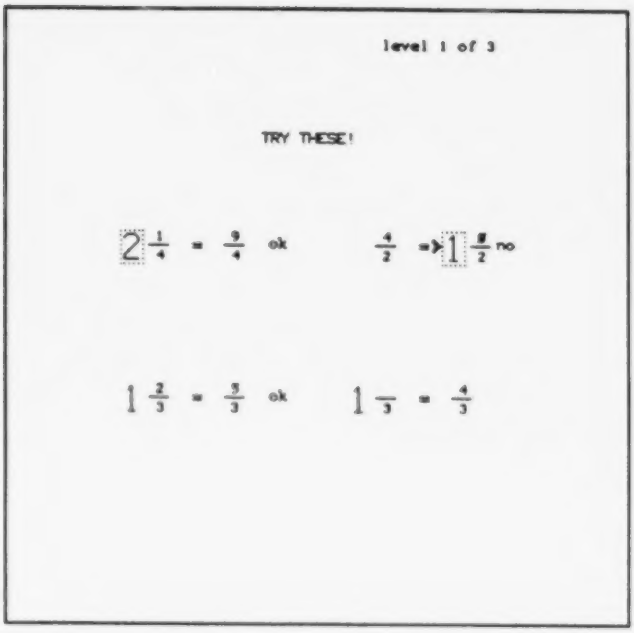



**Figure 1**

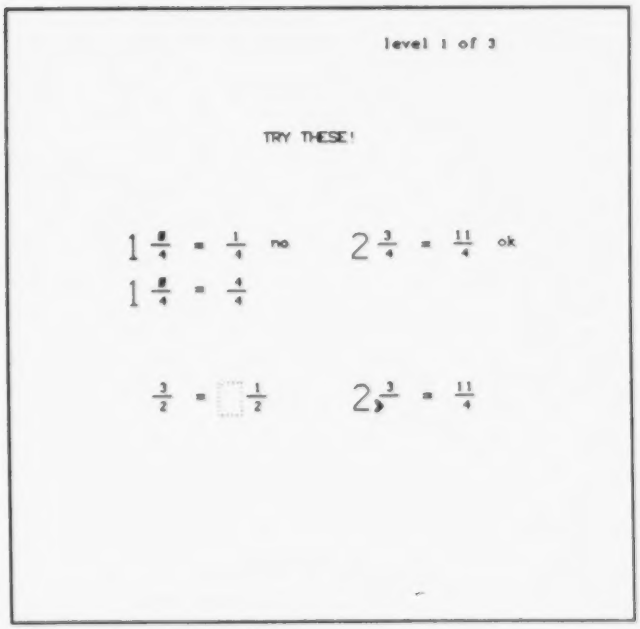



**Figure 2**

**Lesson** **Try These: Equivalent Mixed Numbers**

**File** *tryeq (SYS 4 file name: tryeq.5)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To rename mixed numbers.

**Description** As in the previous lesson, *Try These: Fractions and Mixed Numbers*, students work with four problems per page and fill in the blanks for missing numerators, denominators, and whole numbers (see Figure 1). In this version, students convert one mixed number to another.

**Interaction**

- If students answer incorrectly twice, the computer gives the correct answer (see Figure 2).
- Items are generated until students reach mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons**
*Try These: Fractions and Mixed Numbers*
*Checkup: Equivalent Mixed Numbers*
*Try These: Equivalent Fractions*
*Checkup: Try These Equivalent Fractions*

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

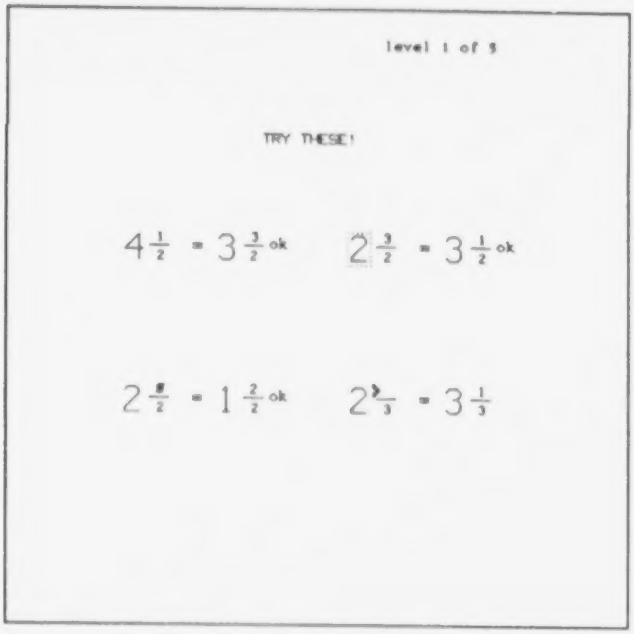



**Figure 1**

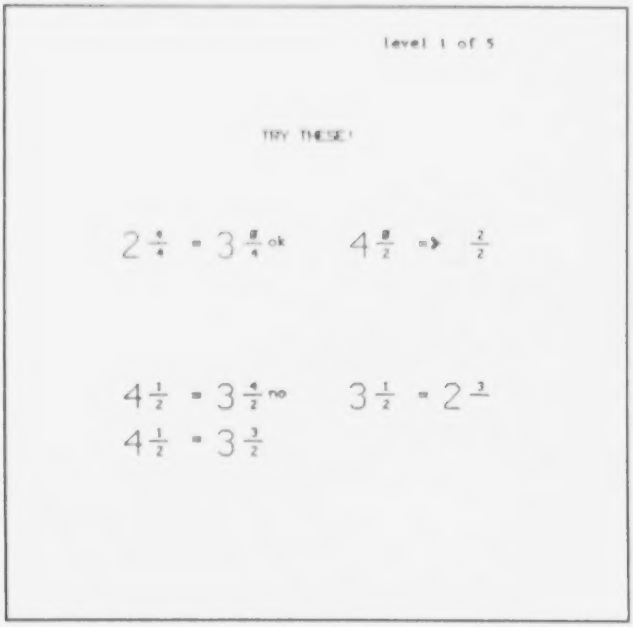



**Figure 2**

**Lesson** **Vocabulary: Whole Number, Fraction, Mixed Number**

(*SYS 4 catalog name*: **Vocabulary: Whole #, Fraction, Mixed #**)

**File** *nlintroc (SYS 4 file name: nlintroc.3)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To learn the distinctions among the concepts: *fractions, whole numbers*, and ***mixed numbers***.

**Description** Students demonstrate their knowledge of whole numbers, fractions, and mixed numbers by providing examples of each type (see Figure 1).

**Interaction**
- Students interact with on-screen dialog and provide their own examples.
- The on-screen dialog helps pinpoint the distinctions between the different categories of number forms.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (self-paced)

**Associated Lesson** *Sort Numbers by Form*

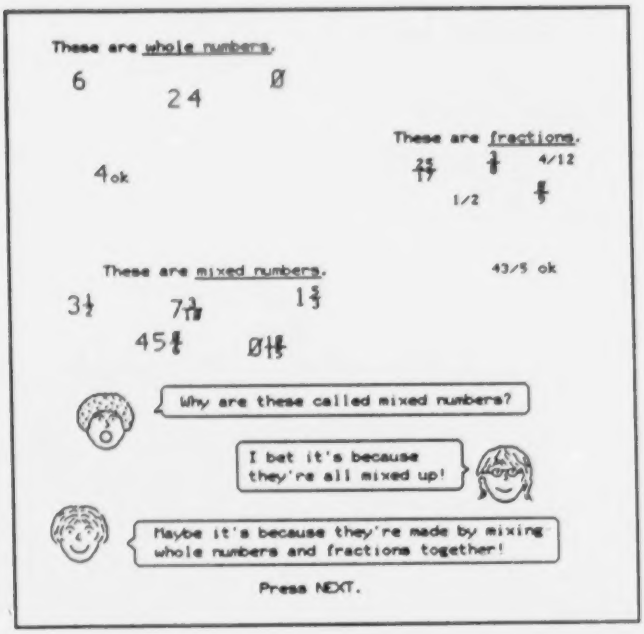

These are whole numbers.
6
24
Ø
4ok
These are fractions.
4/12
1/2
These are mixed numbers.
43/5 ok
Why are these called mixed numbers?
I bet it's because they're all mixed up!
Maybe it's because they're made by mixing whole numbers and fractions together!
Press NEXT.

**Figure 1**

**Lesson** **Sort Numbers by Form**

**File** *sortf (SYS 4 file name: sortf.1)*

**Authors** Sharon Dugdale
David Kibbey
Helen Lueng

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To distinguish among fractions, whole numbers, and mixed numbers.

**Description** Students are presented with a variety of fractions, whole numbers, and mixed numbers, which they are to sort into appropriate boxes (see Figure 1).

**Interaction**

- Students touch the number they want to move, and then, touch the box it should be in.
- Numbers can be moved from one box to another, if students change their minds.
- When done, students ask the computer to check their work.
- If some of the numbers are not properly placed, students are given a chance to reorganize them (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lesson** *Vocabulary: Whole Number, Fraction, Mixed Number*

**Special Note** Touch capability required

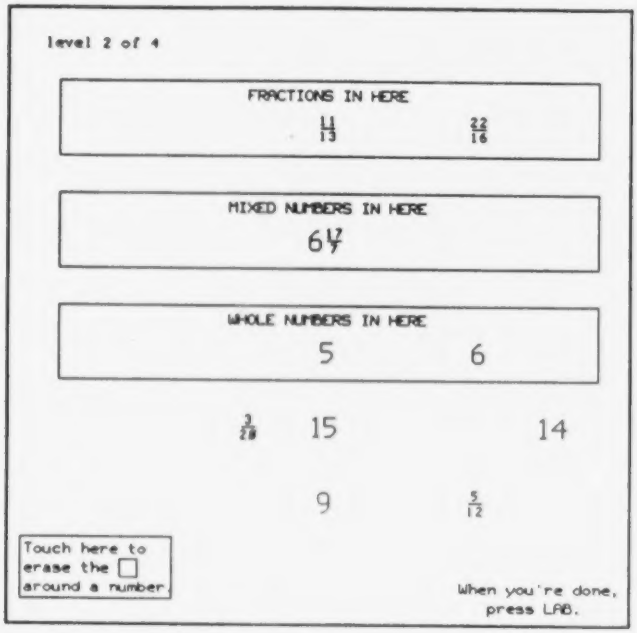



**Figure 1**

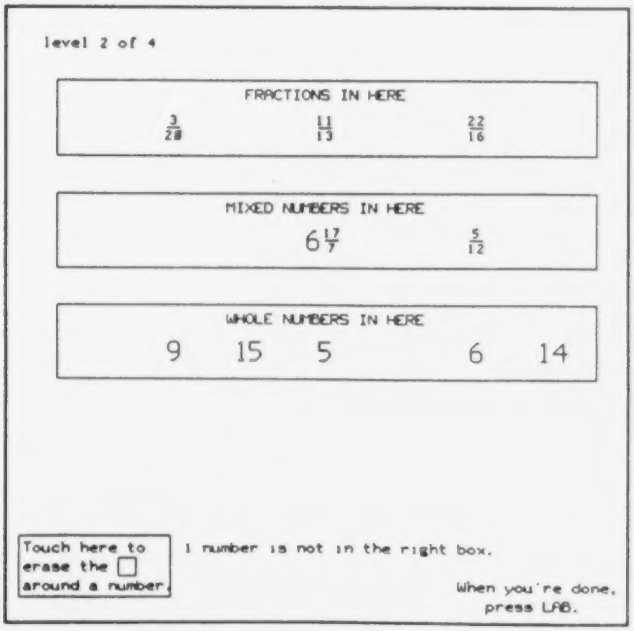



**Figure 2**

**Lesson** **Checkup: Mixed Numbers with Pizza**

**File** *pmix (SYS 4 file name: pmix,9)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**
- To convert a fraction to a mixed number.
- To convert a mixed number to a fraction.

**Description** Students are tested on the concepts presented in *Pizza: Mixed Number Introduction.*

**Interaction**
- Students convert mixed numbers to fractions and vice versa (see Figure 1).
- Problems are presented with and without pictures.
- If errors are made, feedback guides students to the right answer.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (self-paced)

**Associated Lessons** *Pizza: Mixed Number Introduction*
*Pizza: Common Denominator*

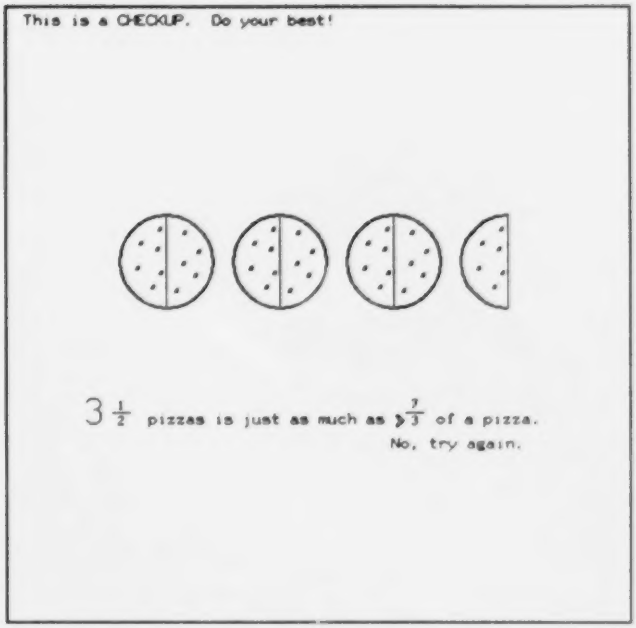

This is a CHECKUP. Do your best!
3 1/2 pizzas is just as much as > 7/3 of a pizza.
No, try again.

**Figure 1**

**Lesson** ## Checkup: Equivalent Mixed Numbers

**File** *tryeq (SYS 4 file name: tryeq.8)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice renaming mixed numbers.

**Description** As in the earlier lesson, *Try These: Equivalent Mixed Numbers*, students work with four problems per page and fill in the blanks for missing numerators, denominators, and whole numbers (see Figure 1). These problems are from the higher levels of the earlier lesson.

**Interaction**

- If students answer incorrectly twice, the computer gives the correct answer (see Figure 2).
- Items are generated until students reach mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons**
*Try These: Fractions and Mixed Numbers*
*Try These: Equivalent Mixed Numbers*
*Try These: Equivalent Fractions*
*Checkup: Try These: Equivalent Fractions*

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

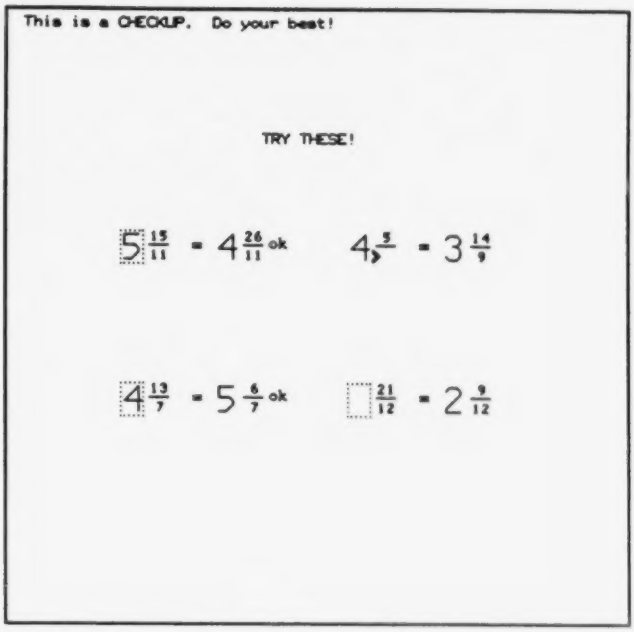



**Figure 1**

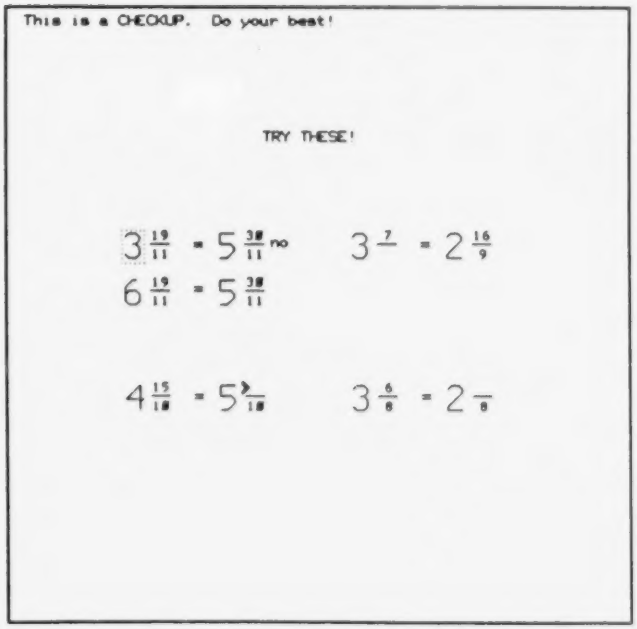



**Figure 2**

**Lesson** **Lights: Equivalent Fractions 2**

**File** *liteseq (SYS 4 file name: liteseq.2)*

**Authors** Sharon Dugdale
David Kibbey
Susan Frasier Lamprinos
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To further develop the concept of equivalent fractions.

**Description** Students build on what they learned in *Lights: Equivalent Fractions 1.* They recognize equivalent fractions by observing, for example, that 7 out of 14 light bulbs are the same as 1/2 (see Figure 1). Items are more difficult at this level.

**Interaction**

- Students touch the appropriate number of light bulbs to construct a representation of the target fraction.
- Students tell how many bulbs they turned on to achieve the equivalent fraction requested. Later, they must also tell how many lights in the set (i.e., give both numerator and denominator).
- Light bulbs appear in arrays, making visualization of the equivalencies easier.
- If students make a mistake in determining the equivalent fraction, they are guided toward the correct answer (see Figure 2).
- Lesson difficulty adjusts to student performance.
- Items are generated until students achieve mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons** *Lights*
*Checkup: Lights*
*Lights: Equivalent Fractions 1*

**Special Note** Touch capability required

Turn on $\frac{1}{4}$ of the lights.
When you are done, press LAB.
Hint: There are 4 columns.
Turn on all the lights in 1 column.
level 4 of 10

**Figure 1**

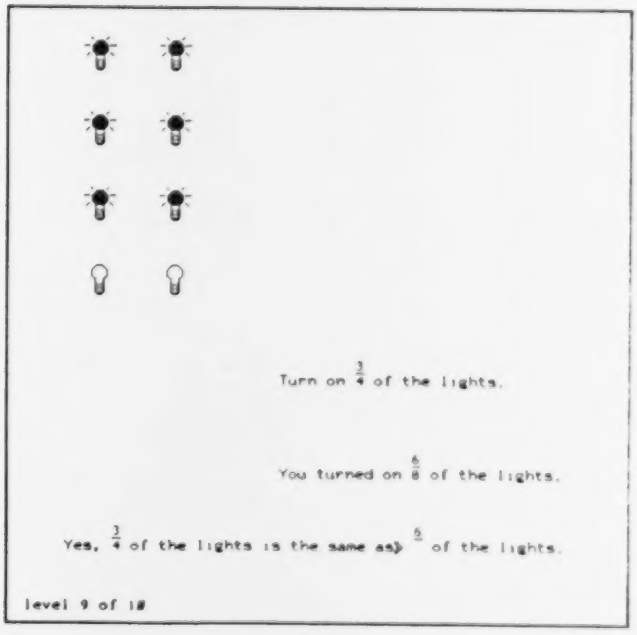

Turn on $\frac{3}{4}$ of the lights.
You turned on $\frac{6}{8}$ of the lights.
Yes, $\frac{3}{4}$ of the lights is the same as $\frac{6}{8}$ of the lights.
level 9 of 10

**Figure 2**

**Lesson** **Boxes: Name Equivalent Fractions**

**File** *recnams (SYS 4 file name: recnames.1)*

**Authors** Sharon Dugdale
David Kibbey
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**
- To name equivalent fractions.
- To recognize equivalent fractions pictorially.

**Description** Students name fractions (with a denominator less than 50) equivalent to the fraction given by the computer. An associated picture is produced for the original fraction and for all the fractions the students name (see Figure 1).

**Interaction**
- Students are allowed to double check their accuracy before being judged by the computer.
- Students are given feedback in the form of pictures of their fractions.
- If some of the fractions are not equivalent, students are asked to find them and change them (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons** *Sort Equivalent Fractions*
*Try These: Equivalent Fractions*

**Special Note** Touch capability required

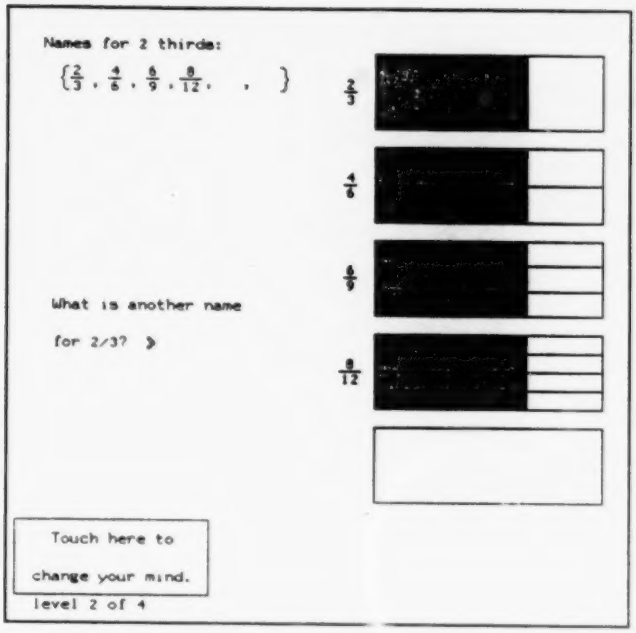



**Figure 1**

Names for 2 fourths:
{2/4 , 3/5 , 4/5 , 4/8 , 6/12 , 1/2 }
Are all the fractions
names for 2/4? n
Touch the boxes you think
are wrong. When you're done,
press LAB.
level 2 of 4

**Figure 2**

**Lesson** **Sort Equivalent Fractions**

**File** *sorteqf (SYS 4 file name: sorteqf.1)*

**Authors** Sharon Dugdale
David Kibbey
Helen Lueng

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To identify equivalent fractions.

**Description** Students are presented with a variety of fractions, which they sort into appropriate loops containing the equivalent fraction.

**Interaction**

- Students touch the fraction they want to move, and then, touch the loop it should be in (see Figure 1).
- Fractions can be moved from one loop to another, if students change their minds.
- When done, students ask the computer to check their work.
- If some of the fractions are not in the correct loop, students are told how many are not properly placed (see Figure 2).
- Students are then given a chance to reorganize the items, and they will be checked again.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons** *Sort Equivalent Mixed Numbers*
*Checkup: Sort Equivalent Fractions*
*Boxes: Name Equivalent Fractions*
*Sort Number Names with Decimals*

**Special Note** Touch capability required

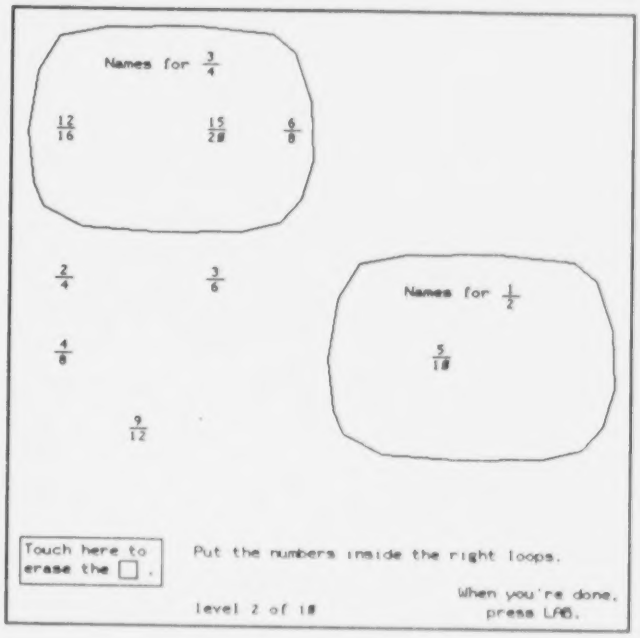



**Figure 1**

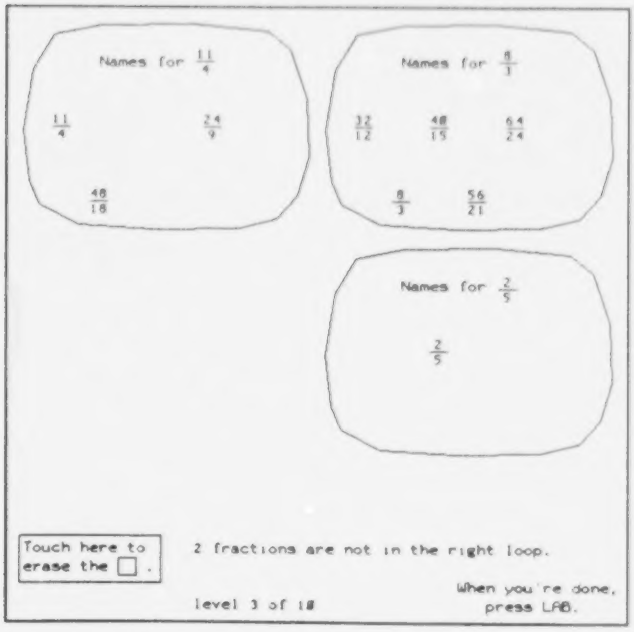



**Figure 2**

**Lesson** **Sort Equivalent Mixed Numbers**

**File** *sorteqm (SYS 4 file name: sorteqm.1)*

**Authors** Sharon Dugdale
David Kibbey
Helen Lueng

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To distinguish equivalent mixed numbers.

**Description** Students are presented with a variety of mixed numbers, which they sort into appropriate loops containing the equivalent mixed number.

**Interaction**

- Students touch the number they want to move, and then, touch the loop it should be in (see Figure 1).
- Numbers can be moved from one loop to another, if students change their minds.
- When done, students ask the computer to check their work.
- If some of the numbers are not properly placed, students are given a chance to reorganize them (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 18 minutes (self-paced)

**Associated Lessons** *Sort Equivalent Fractions*
*Checkup: Sort Equivalent Fractions*

**Special Note** Touch capability required

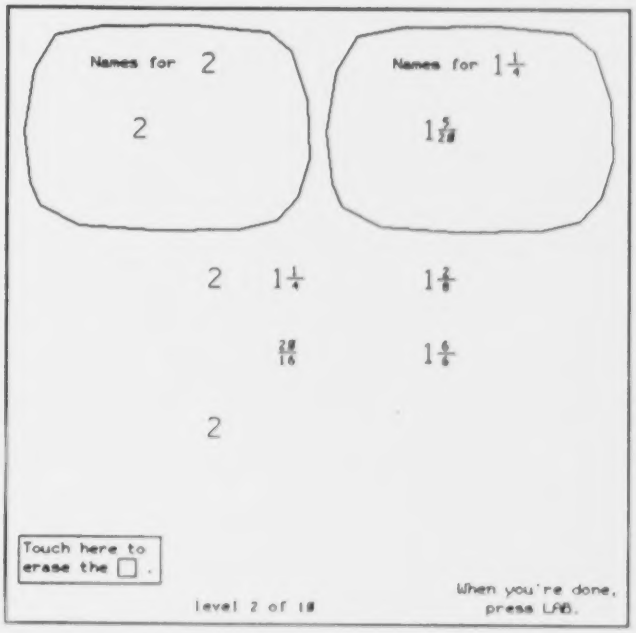



**Figure 1**

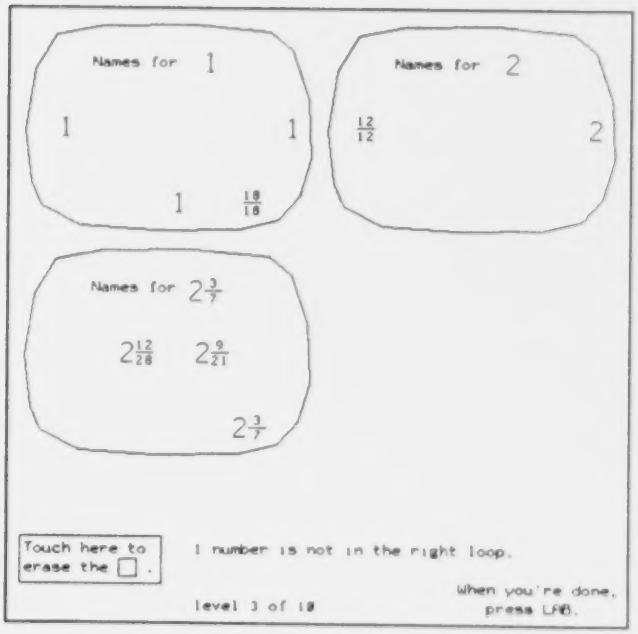



**Figure 2**

**Lesson** ## Try These: Equivalent Fractions

**File** *tryeq (SYS 4 file name: tryeq.1)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To rename fractions.

**Description** As in two earlier lessons of this type, students work with four problems per page and fill in the blanks for missing numerators, denominators, and whole numbers (see Figure 1). In this version, students convert one fraction to another.

**Interaction**

- If students answer incorrectly twice, the computer gives the correct answer (see Figure 2).
- Lesson difficulty adjusts to student performance.
- Items are generated until students achieve mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons**
*Try These: Fractions and Mixed Numbers*
*Try These: Equivalent Mixed Numbers*
*Checkup: Equivalent Mixed Numbers*
*Checkup: Try These: Equivalent Fractions*
*Boxes: Name Equivalent Fractions*

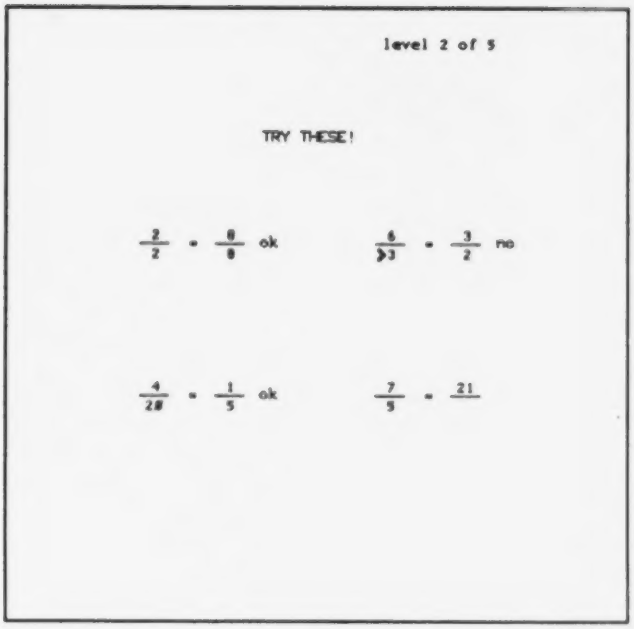



**Figure 1**

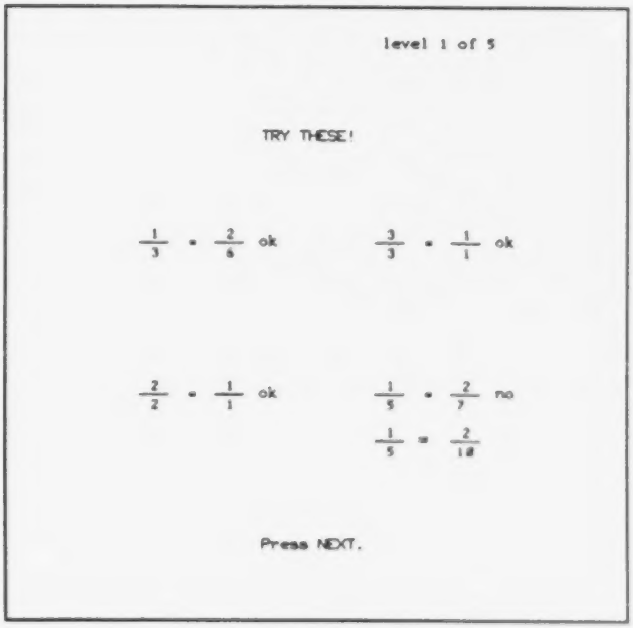



**Figure 2**

**Lesson** **Introduction to Simplest Terms**

**File** *sti (SYS 4 file name: sti)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To learn to recognize when a fraction is in simplest terms.

**Description** *Simplest terms* is defined as the name of a fraction with the smallest numerator and denominator. Students choose the simplest term name from among the set of equivalent fractions given.

**Interaction**

- Students are shown several sets of equivalent fractions.
- They are told that the name with the smallest numerator and denominator is the *simplest terms* name (see Figure 1).
- With the sets still on the screen, students are asked to pick out the simplest terms name for a series of fractions (selected from the sets shown). (See Figure 2.)
- If students make errors, they are guided toward the correct answer (see Figures 3 and 4).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Checkup: Simplest Terms*
*Simplest Terms: Equivalent Fractions*

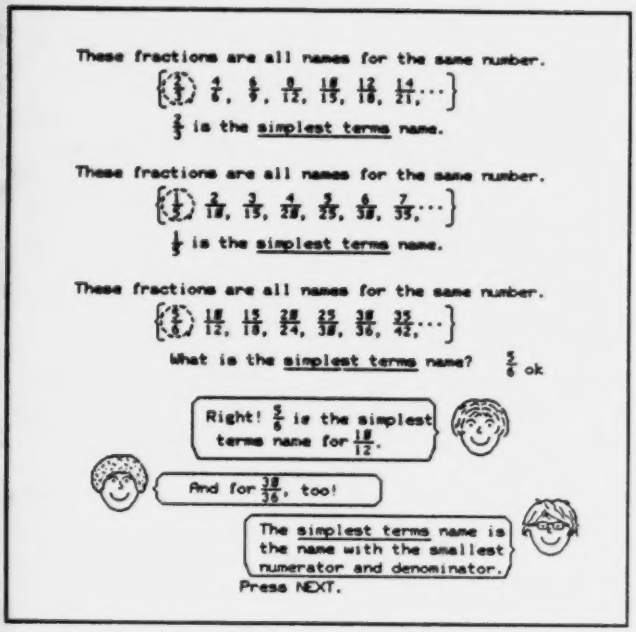

These fractions are all names for the same number.
$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \cdots\right\}$
$\frac{2}{3}$ is the simplest terms name.
These fractions are all names for the same number.
$\left\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \frac{7}{35}, \cdots\right\}$
$\frac{1}{5}$ is the simplest terms name.
These fractions are all names for the same number.
$\left\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36}, \frac{35}{42}, \cdots\right\}$
What is the simplest terms name? $\frac{5}{6}$ ok
Right! $\frac{5}{6}$ is the simplest terms name for $\frac{10}{12}$.
And for $\frac{30}{36}$, too!
The simplest terms name is the name with the smallest numerator and denominator.
Press NEXT.

Figure 1

Here are some equivalence sets.
$\left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{20}{25}, \frac{24}{30}, \frac{28}{35}, \cdots\right\}$
$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \frac{7}{21}, \cdots\right\}$
$\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \frac{7}{28}, \cdots\right\}$
What is the simplest terms name for $\frac{6}{10}$? $\frac{1}{5}$ ok
What is the simplest terms name for $\frac{4}{16}$? $\frac{1}{4}$ ok
What is the simplest terms name for $\frac{8}{10}$? $\frac{4}{5}$ ok
What is the simplest terms name for $\frac{7}{21}$? >

Figure 2

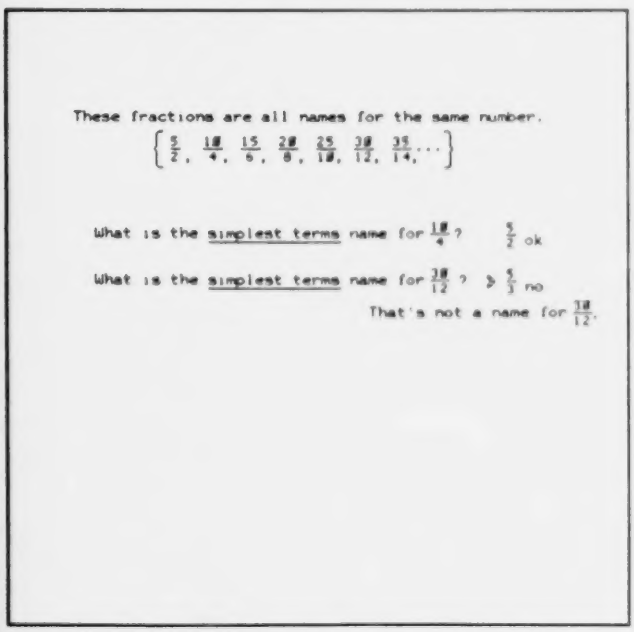

These fractions are all names for the same number.
$\left\{\frac{5}{2}, \frac{10}{4}, \frac{15}{6}, \frac{20}{8}, \frac{25}{10}, \frac{30}{12}, \frac{35}{14}, \cdots\right\}$
What is the simplest terms name for $\frac{10}{4}$? $\frac{5}{2}$ ok
What is the simplest terms name for $\frac{30}{12}$? > $\frac{5}{3}$ no
That's not a name for $\frac{30}{12}$.

Figure 3

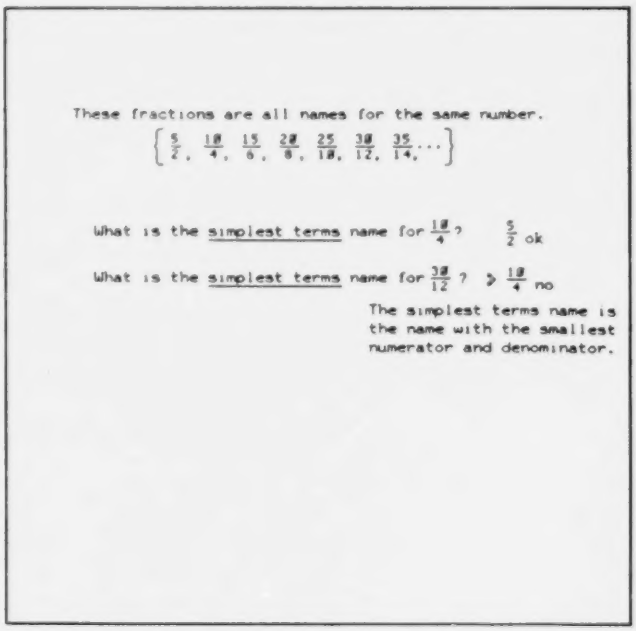

These fractions are all names for the same number.
$\left\{\frac{5}{2}, \frac{10}{4}, \frac{15}{6}, \frac{20}{8}, \frac{25}{10}, \frac{30}{12}, \frac{35}{14}, \cdots\right\}$
What is the simplest terms name for $\frac{10}{4}$? $\frac{5}{2}$ ok
What is the simplest terms name for $\frac{30}{12}$? > $\frac{10}{4}$ no
The simplest terms name is the name with the smallest numerator and denominator.

Figure 4

**Lesson** **Pizza: Common Denominator**

**File** *pcd (SYS 4 file name: pcd.1)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To find a common denominator among fractions.
- To find equivalent fractions, given a common denominator.

**Description** Students practice finding common denominators and converting fractions. Pictures help students visualize the problem.

**Interaction**

- Students distribute pizzas and partial pizzas among two or three persons who have requested certain amounts of pizza (see Figure 1).
- Students must find common denominators and convert the requests to fill the order.
- If errors are made, feedback guides students to the right answer (see Figure 2).
- Help is available for finding a common denominator (see Figure 3).
- Lesson difficulty adjusts to student performance.
- Problems are generated until students reach mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons** *Pizza: Mixed Numbers Introduction*
*Checkup: Mixed Numbers with Pizza*

**Special Note** Touch capability required

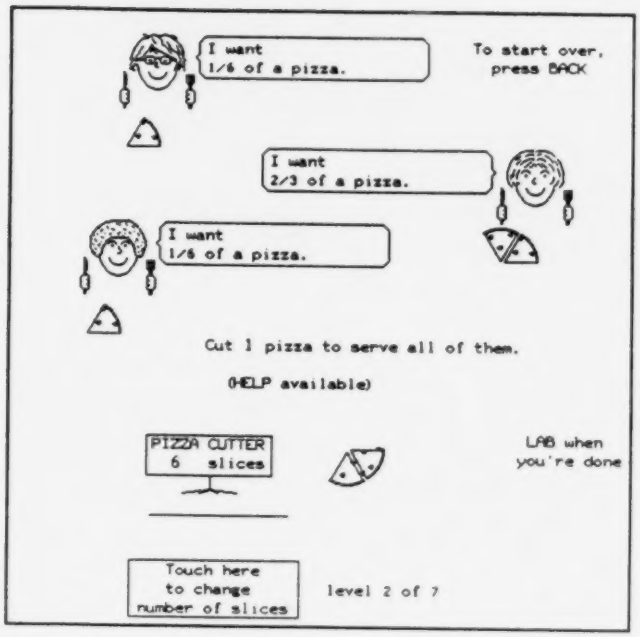



**Figure 1**

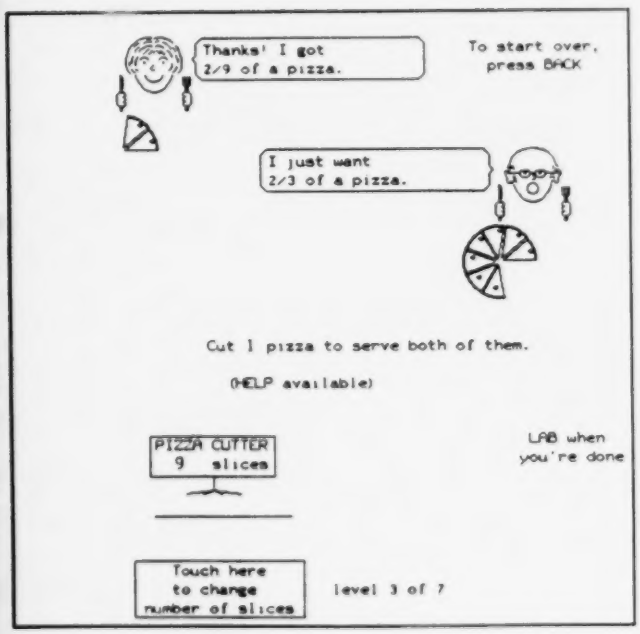



**Figure 2**

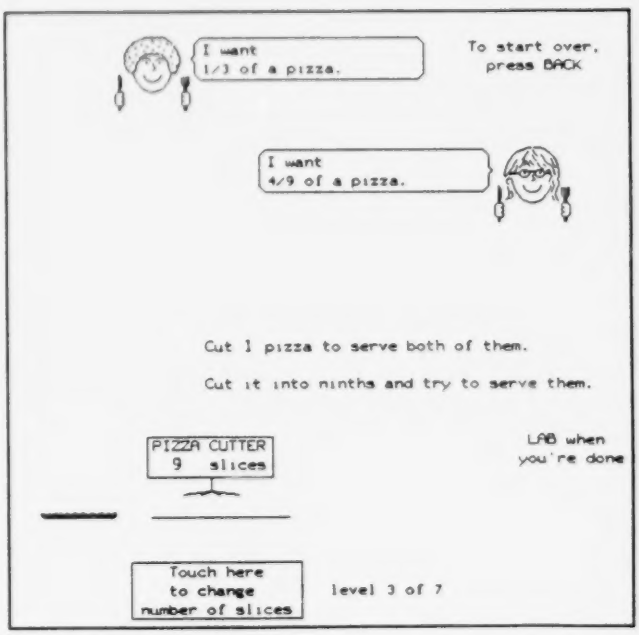



**Figure 3**

**Lesson** ## Checkup: Sort Equivalent Fractions

**File** *sorteqf (SYS 4 file name: sorteqf.3)*

**Authors** Sharon Dugdale
David Kibbey
Helen Lueng

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice identifying equivalent fractions.

**Description** As in the earlier lesson, *Sort Equivalent Fractions*, students are presented with a variety of fractions, which they sort into appropriate loops containing equivalent fractions.

**Interaction**

- Students touch the number they want to move, and then, touch the loop it should be in (see Figure 1).
- Numbers can be moved from one loop to another, if students change their minds.
- When done, students ask the computer to check their work.
- If some of the numbers are not properly placed, students are given a chance to reorganize them (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (self-paced)

**Associated Lessons** *Sort Equivalent Fractions*
*Sort Equivalent Mixed Numbers*

**Special Note** Touch capability required

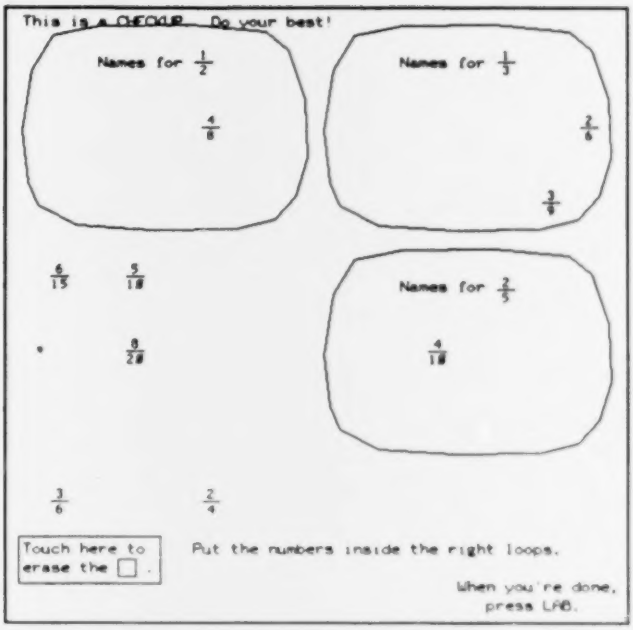



**Figure 1**

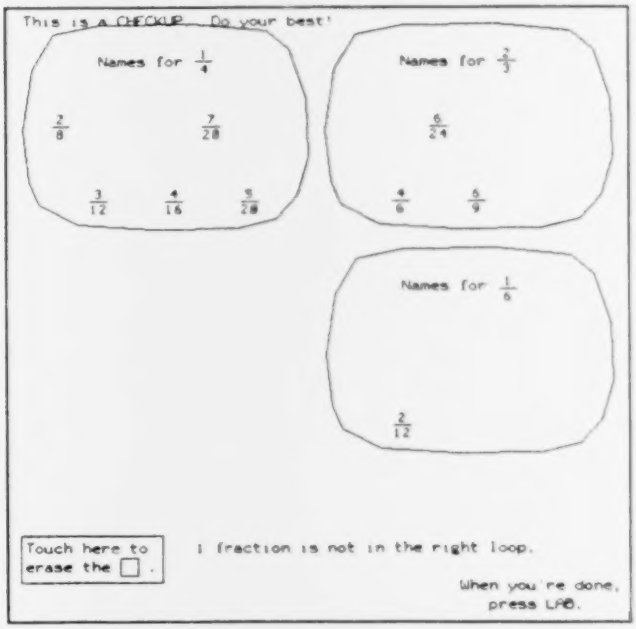



**Figure 2**

**Lesson** ## Checkup: Try These Equivalent Fractions

**File** *tryeq (SYS 4 file name: tryeq.7)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To rename fractions.

**Description** Students are tested on what they learned in *Try These: Equivalent Fractions*. They work with four problems per page and fill in the blanks for missing numerators, denominators, and whole numbers (see Figure 1). These problems are similar to those at the higher levels in the earlier lesson.

**Interaction**

- If students answer incorrectly twice, the computer gives the correct answer (see Figure 2).
- Items are generated until students reach mastery.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons**
*Try These: Fractions and Mixed Numbers*
*Try These: Equivalent Mixed Numbers*
*Checkup: Equivalent Mixed Numbers*
*Try These: Equivalent Fractions*

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

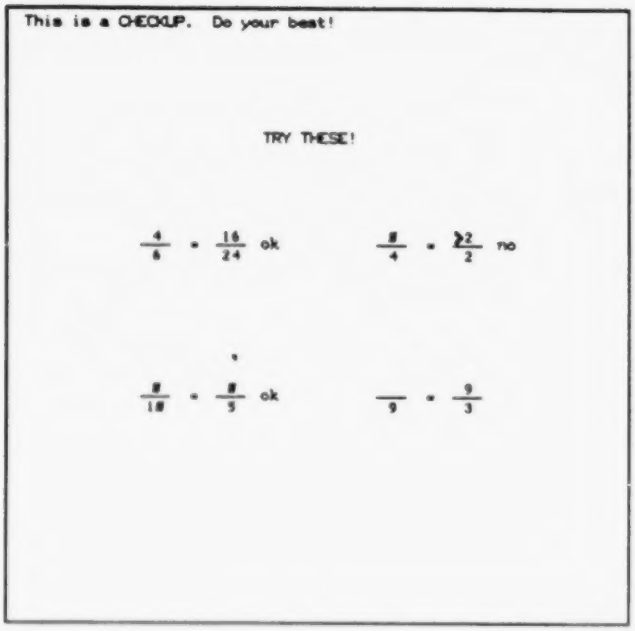



**Figure 1**

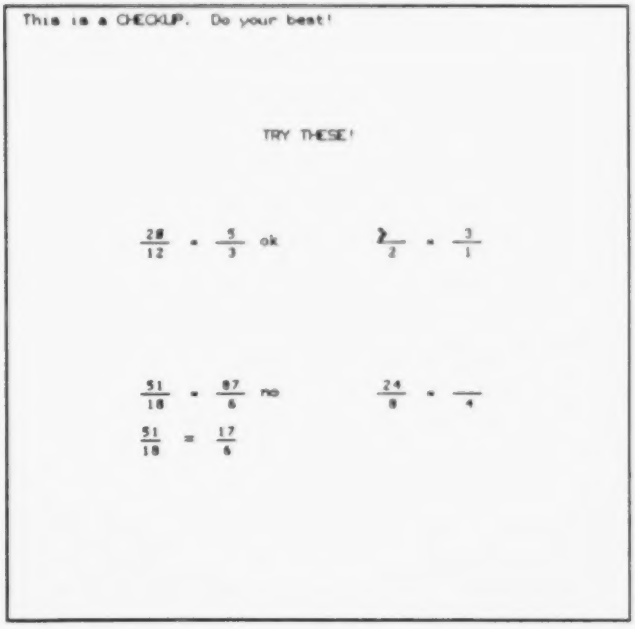



**Figure 2**

**Lesson** ## Checkup: Simplest Terms

**File** *sterms (SYS 4 file name: sterms.4)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To reduce fractions to simplest terms.

**Description** As in the earlier lesson, *Introduction to Simplest Terms*, students write simplest term names for given fractions.

**Interaction**

- Students are asked for the simplest terms name of the fraction given (see Figure 1).
- If errors are made, students are guided to the correct answer (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (self-paced)

**Associated Lessons** *Introduction to Simplest Terms*
*Simplest Terms: Equivalent Fractions*

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

**Figure 1**

This is a CHECKUP. Do your best!
What is the simplest terms name for 18/38?
> 9/15 no
That's a name for 18/38,
but it's not in simplest terms.

**Figure 2**

# Unit 11

## Adding and Subtracting Fractions with Like Denominators

**Objective** To add and subtract fractions with like denominators.

**Description** Students are taught mixed number notation, adding and subtracting mixed numbers, and simplifying answers to standard form. Borrowing is also taught and practiced.

**Lessons**

- Mixed Number Notation
  Mixed Number Answer Format
  Add and Subtract Mixed Numbers
  Simplifying Answers to Addition Problems
  Standard Form Practice
  Addition Practice, Simplifying Answers
  Subtraction with Borrowing
  Addition and Subtraction Drill [Like Denominators]
  Addition and Subtraction with Simplifying and Borrowing
  Checkup: Standard Form
  Checkup: Add and Subtract Mixed Numbers

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | Adding Fractions -- Like Denominators |
| 3A, | 3B, | 3C | Adding Mixed Numbers -- Like Denominators |
| 5A, | 5B, | 5C | Subtracting Fractions -- Like Denominators |
| 7A, | 7B, | 7C | Subtracting Fractions from the Whole Number |
| 9A, | 9B, | 9C | Subtracting Mixed Numbers -- With and Without Borrowing |

**Lesson** **Mixed Number Notation**

**File** *nlintrob (SYS 4 file name: nlintrob.1)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To introduce the typing of mixed numbers.
- To practice locating and writing mixed numbers on a number line.

**Description** This lesson shows students a number line that extends from *0* to *3*. They fill in the fractions at the indicated spaces (see Figure 1).

**Interaction**

- Students type in the fraction requested.
- If students miss an answer, they are shown exactly what to type.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 2 minutes

**Associated Lesson** *Mixed Number Answer Format*

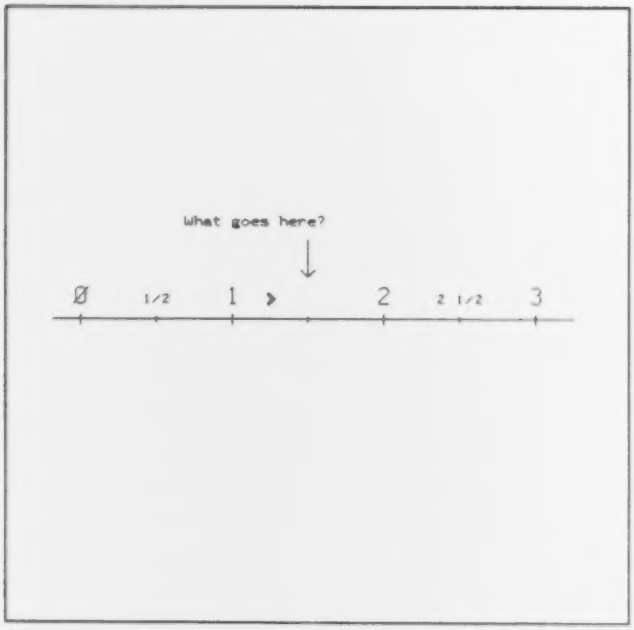



**Figure 1**

**Lesson** **Mixed Number Answer Format**

(*SYS 4 catalog name*: **New Answer Format**)

**File** *naf (SYS 4 file name: naf.1)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To show two ways fractions and mixed numbers can appear on the screen.

**Description** Students are shown two ways that fractions and mixed numbers appear (see Figure 1).

**Interaction** Students follow the directions on the screen as they type fractions and mixed numbers. The program chooses which way the numbers will be displayed.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 2 minutes

**Associated Lesson** *Mixed Number Notation*

Write $2\frac{1}{2}$.
2 1/2 ok
Write $2\frac{1}{2}$ again.
(It will look different.)
$2\frac{1}{2}$ ok
Write $4\frac{2}{3}$.
$4\frac{2}{3}$ ok
Sometimes your answer will look like this.
Write $4\frac{2}{3}$ again.
4 2/3 ok
Sometimes it will look like this.
HELP Available.

**Figure 1**

**Lesson** **Add and Subtract Mixed Numbers**

(*SYS 4 catalog name*: **Adding and Subtracting Mixed Numbers**)

**File** *pad1 (SYS 4 file name: pad1.1)*

**Authors** Sharon Dugdale
David Kibbey
Barry Cohen

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To add and subtract mixed numbers.

**Description** Students practice adding and subtracting mixed numbers. At lower levels, fractions are represented by pizzas to help students visualize the numbers they are working with. At higher levels, pictures are available only if help is requested.

Students begin at Level 1, adding and subtracting one whole number and one fraction. By Level 10, they are adding and subtracting two mixed numbers. They do not need to carry or borrow.

**Interaction**

- Students type in the answers to the questions given (see Figure 1).
- If students answer a problem incorrectly three times, a picture of the problem automatically appears (if it was not there to start with).
- Feedback appropriate to common errors is given (see Figure 2).
- Correct answers bring harder problems; missed items bring easier problems.
- Problems are generated until students reach mastery.
- The lesson is completed after Level 10. If students choose to do more problems, they return to Level 11 and receive a variety of problem types.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 25 minutes (self-paced)

**Associated Lessons** *Simplifying Answers to Addition Problems*
*Standard Form Practice*
*Addition Practice, Simplifying Answers*
*Checkup: Standard Form*

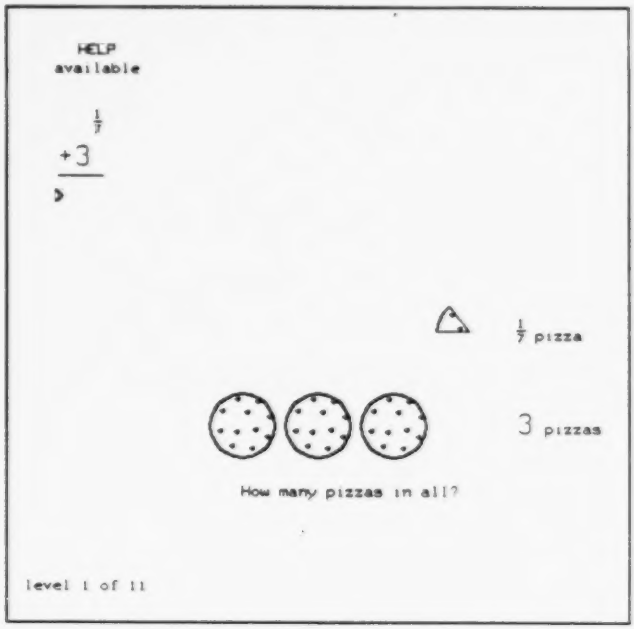



**Figure 1**

**Figure 2**

**Lesson** **Simplifying Answers to Addition Problems**

**File** *pad1 (SYS 4 file name: pad1.3)*

**Authors** Sharon Dugdale
David Kibbey
Barry Cohen

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To write fractions and mixed fractions in standard (simplified) form in addition problems.

**Description** This lesson demonstrates that there are many equivalent ways to name fractions and mixed numbers (see Figure 1), but that only one way is the *simplest*. It is called "standard form" (see Figure 2).

**Interaction**

- Students go through an interactive explanation of equivalent names and standard form.
- Students practice writing fractions and mixed numbers in standard form (see Figure 3).
- If students answer incorrectly three times, the answer is given.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons**
*Add and Subtract Mixed Numbers*
*Standard Form Practice*
*Addition Practice, Simplifying Answers*
*Checkup: Standard Form*

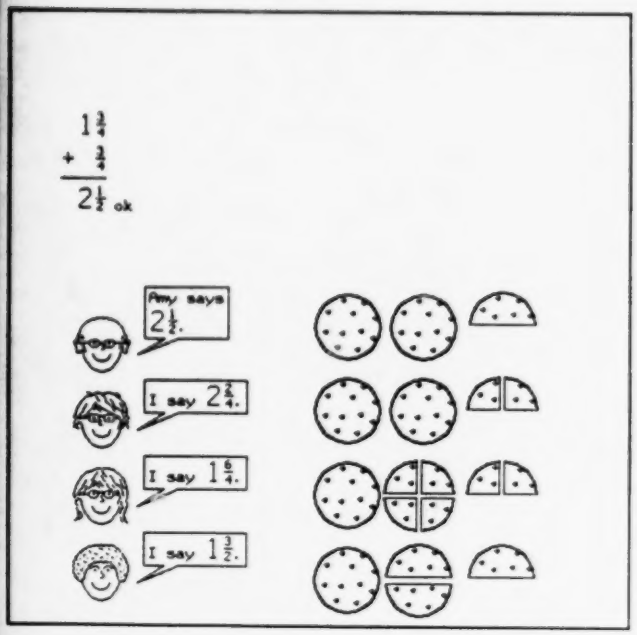



**Figure 1**

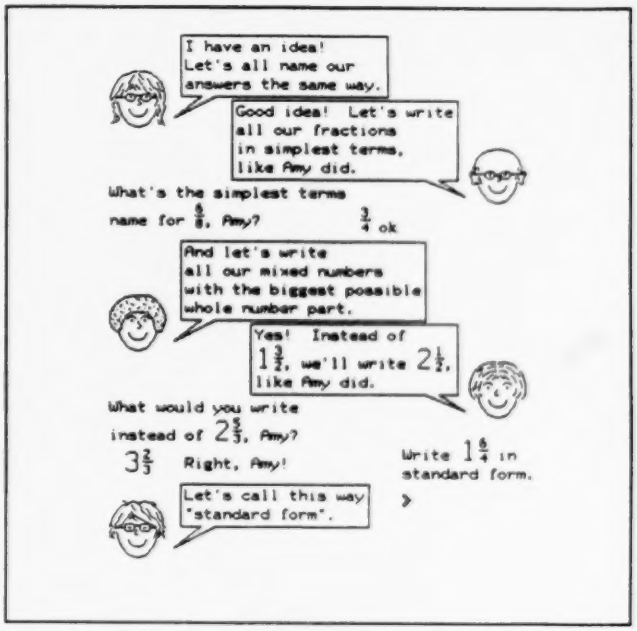

I have an idea! Let's all name our answers the same way.
Good idea! Let's write all our fractions in simplest terms, like Amy did.
What's the simplest terms name for 6/8, Amy?
3/4 ok
And let's write all our mixed numbers with the biggest possible whole number part.
Yes! Instead of 1 3/2, we'll write 2 1/2, like Amy did.
What would you write instead of 2 5/3, Amy?
3 2/3 Right, Amy!
Write 1 6/4 in standard form.
Let's call this way "standard form".

**Figure 2**

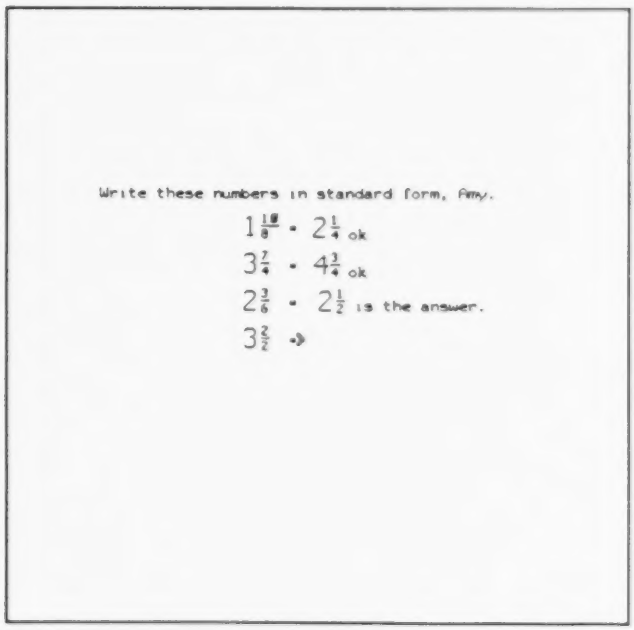



**Figure 3**

**Lesson** **Standard Form Practice**

**File** *pad1 (SYS 4 file name: pad1,4)*

**Authors** Sharon Dugdale
David Kibbey
Barry Cohen

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice writing fractions and mixed numbers in standard form.

**Description** Students are presented with fractions and mixed numbers to write in standard form (see Figure 1). Sometimes the number already is in standard form.

**Interaction**

- Students are given four problems on a page.
- After a third incorrect answer on any one problem, the correct answer is given.
- Students work until mastery level is achieved.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons** *Add and Subtract Mixed Numbers*
*Simplifying Answers to Addition Problems*
*Addition Practice, Simplifying Answers*
*Checkup: Standard Form*

level 2 of 3
Standard form name uses biggest whole number possible and fraction in simplest terms.
Write the standard form name for these numbers:
$1\frac{2}{4}$ • $1\frac{1}{2}$ ok
$1\frac{3}{8}$ • $1\frac{3}{8}$ ok
$1\frac{2}{7}$ • $1\frac{2}{7}$ ok
$\frac{4}{5}$ -> $\frac{8}{10}$ no
That's a name for $\frac{4}{5}$, but we want the fraction in simplest terms.

**Figure 1**

**Lesson** **Addition Practice, Simplifying Answers**

**File** *pad1 (SYS 4 file name: pad1,6)*

**Authors** Sharon Dugdale
David Kibbey
Barry Cohen

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice adding fractions and mixed numbers and putting the answers in standard form.

**Description** Students add fractions and mixed numbers. Answers must be converted to standard form (see Figure 1).

Students begin by adding mixed numbers and fractions. Most problems require carrying. Higher levels have more difficult fractions to add, the whole number portion remaining single digit (see Figure 2).

**Interaction**

- Students can press HELP to see a picture of the problem in the form of pizzas.
- If students answer incorrectly three times, a picture automatically appears.
- Students must keep trying until they get each answer right.
- Problems are generated until students achieve mastery level.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons**
*Add and Subtract Mixed Numbers*
*Simplifying Answers to Addition Problems*
*Standard Form Practice*
*Checkup: Standard Form*

**Figure 1**

**Figure 2**

**Lesson** **Subtraction with Borrowing**

**File** *pad2 (SYS 4 file name: pad2.1)*

**Authors** Sharon Dugdale
David Kibbey
Barry Cohen
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To subtract fractions and mixed numbers with common denominators, where borrowing from the whole number is required.
- To write subtraction sentences describing the problem

**Description** Students solve subtraction problems having fractions and mixed numbers with common denominators but requiring borrowing from the whole number.

**Interaction**

- Instructions on using the pizza cutter are available upon entry to the lesson (see Figures 1 and 2).
- Students are given a stack of pizzas and a cutter to make slices. Customers ask for a number of pizzas, and students are to fill the order, guessing how many pizzas will be left (see Figure 3).
- After filling the order, students write a number sentence describing the transaction (see Figure 4).
- They check their estimated answer against the picture of the transaction and modify their answer if needed.
- Students are given additional problems until they reach mastery level.
- HELP is available upon entry to the lesson.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lesson** *Addition and Subtraction with Simplifying and Borrowing*
*Checkup: Add and Subtract Mixed Numbers*

**Special Note** Touch capability required

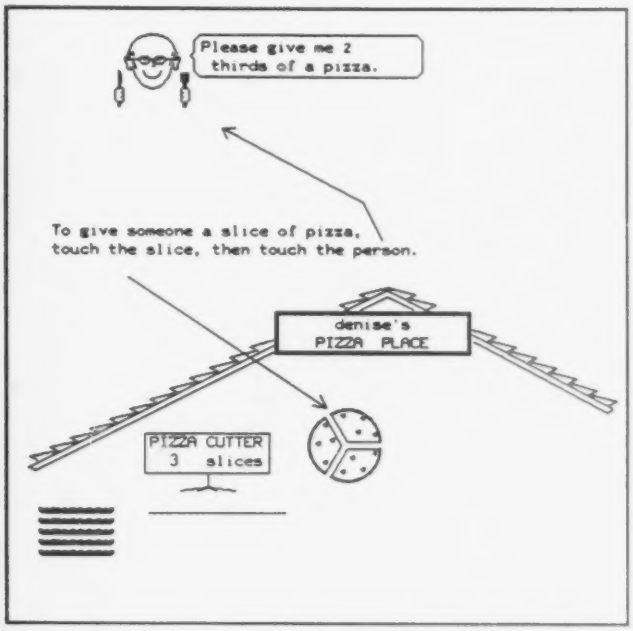

Please give me 2 thirds of a pizza.
To give someone a slice of pizza, touch the slice, then touch the person.
denise's PIZZA PLACE
PIZZA CUTTER
3 slices

**Figure 1**

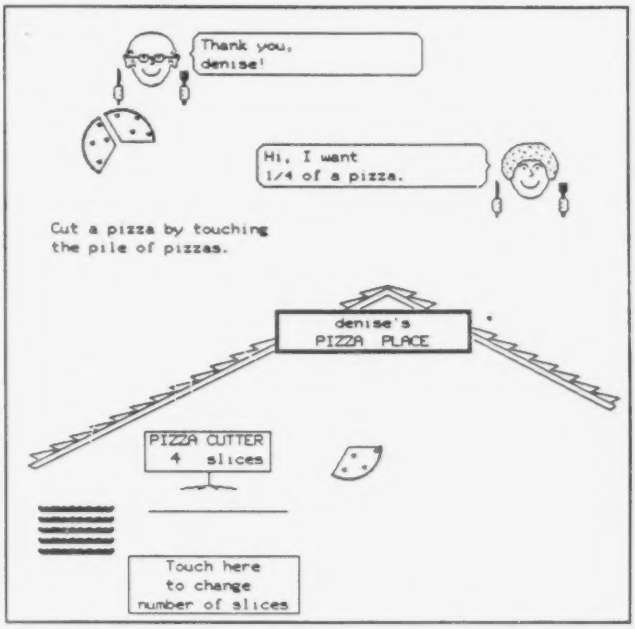

Thank you, denise!
Hi, I want 1/4 of a pizza.
Cut a pizza by touching the pile of pizzas.
denise's PIZZA PLACE
PIZZA CUTTER
4 slices
Touch here to change number of slices

**Figure 2**

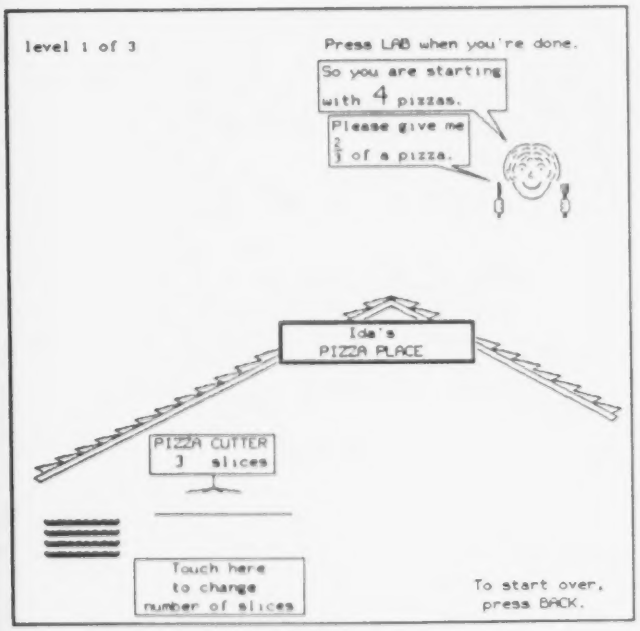

level 1 of 3
Press LAB when you're done.
So you are starting with 4 pizzas.
Please give me 2/3 of a pizza.
Ida's PIZZA PLACE
PIZZA CUTTER
3 slices
Touch here to change number of slices
To start over, press BACK.

**Figure 3**

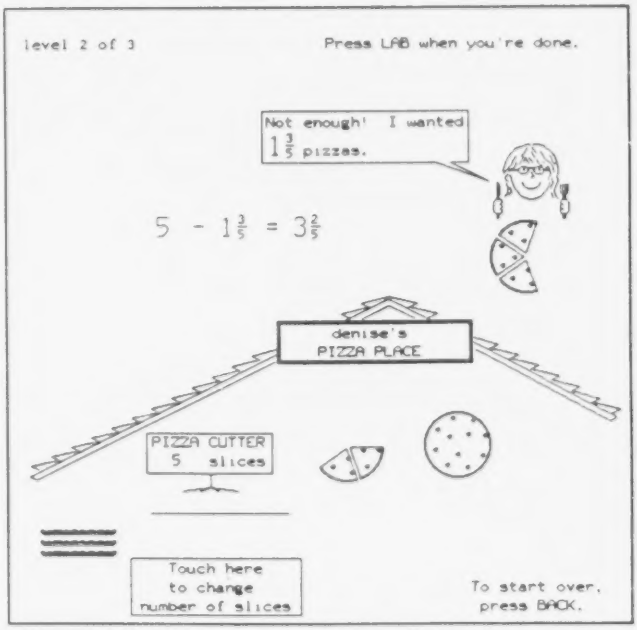

level 2 of 3
Press LAB when you're done.
Not enough! I wanted 1 3/5 pizzas.
5 - 1 3/5 = 3 2/5
denise's PIZZA PLACE
PIZZA CUTTER
5 slices
Touch here to change number of slices
To start over, press BACK.

**Figure 4**

**Lesson** **Addition and Subtraction Drill [Like Denominators]**

(*SYS 4 catalog name*: **Add and Subtract Like Denominators**)

**File** *tryad (SYS 4 file name: tryad,1)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice adding and subtracting fractions and mixed numbers and writing the answer in simplest terms.

**Description** Students practice adding and subtracting fractions and mixed numbers with like denominators. They must put answers in simplest terms (see Figure 1).

**Interaction**

- Students see four addition or subtraction problems per page.
- Special messages appear when students make common errors, such as adding instead of subtracting.
- Lower level problems do not require carrying or borrowing. Higher level problems do.
- On higher levels, problems are presented horizontally as well as vertically.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons** *Addition and Subtraction Drill [Unlike Denominators]*
*Addition and Subtraction with Simplifying and Borrowing*

$\frac{1}{3} - \frac{1}{3} = \emptyset$ ok
$4\frac{1}{12} - \emptyset\frac{1}{12} = 4$ ok
$3\frac{4}{9}$
$- \frac{1}{9}$
$3\frac{3}{9}$
$3\frac{1}{8}$
$+2$
Right number, Amy.
Now write it with the
fraction in simplest terms.
level 7 of 16

**Figure 1**

**Lesson** **Addition and Subtraction with Simplifying and Borrowing**

(*SYS 4 catalog name*: **Simplifying and Borrowing**)

**File** *pad2 (SYS 4 file name: pad2,2)*

**Authors** Sharon Dugdale
David Kibbey
Barry Cohen
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice adding and subtracting fractions and mixed numbers with like denominators when carrying or borrowing is necessary.

**Description** Students are shown pizzas representing numbers. The pizzas are cut into pieces to represent fractions. In order to borrow, a whole pizza is cut into slices, so that enough slices will be available for the subtraction (see Figure 1).

**Interaction**

- Students are led through the steps of borrowing (or carrying) by visualizing the dividing up of pizzas.
- Students then practice the steps without the pictures, although pictures are available if they want or need them (see Figure 2).
- Students practice until they reach mastery level.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons** *Subtraction with Borrowing*
*Checkup: Add and Subtract Mixed Numbers*
*Addition and Subtraction Drill [Like Denominators]*

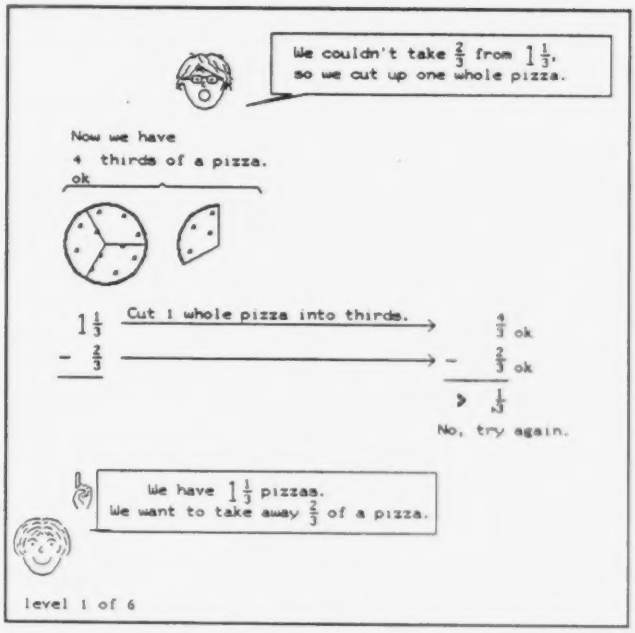

We couldn't take $\frac{2}{3}$ from $1\frac{1}{3}$, so we cut up one whole pizza.
Now we have
4 thirds of a pizza.
ok
$1\frac{1}{3}$ Cut 1 whole pizza into thirds. $\frac{4}{3}$ ok
$-\frac{2}{3}$ $-\frac{2}{3}$ ok
$> \frac{1}{3}$
No, try again.
We have $1\frac{1}{3}$ pizzas.
We want to take away $\frac{2}{3}$ of a pizza.
level 1 of 6

**Figure 1**

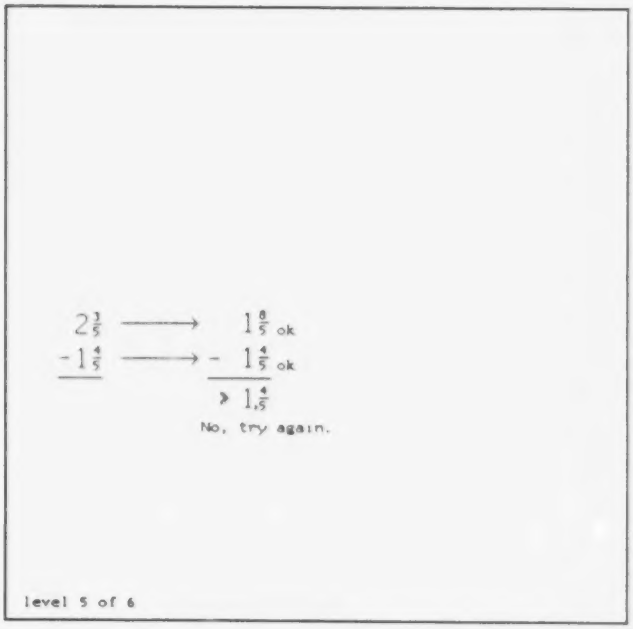



**Figure 2**

**Lesson** **Checkup: Standard Form**

**File** *pad1 (SYS 4 file name: pad1.8)*

**Authors** Sharon Dugdale
David Kibbey
Barry Cohen

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To review converting fractions and mixed numbers to standard form.

**Description** Students rewrite fractions or mixed numbers in standard form (see Figure 1).

**Interaction**

- Problems are presented four at a time.
- If students miss any one problem three times in a row, they are shown the correct answer.
- Students must meet mastery criteria to receive credit for the lesson.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (self-paced)

**Associated Lessons**
*Add and Subtract Mixed Numbers*
*Simplifying Answers to Addition Problems*
*Standard Form Practice*
*Addition Practice, Simplifying Answers*

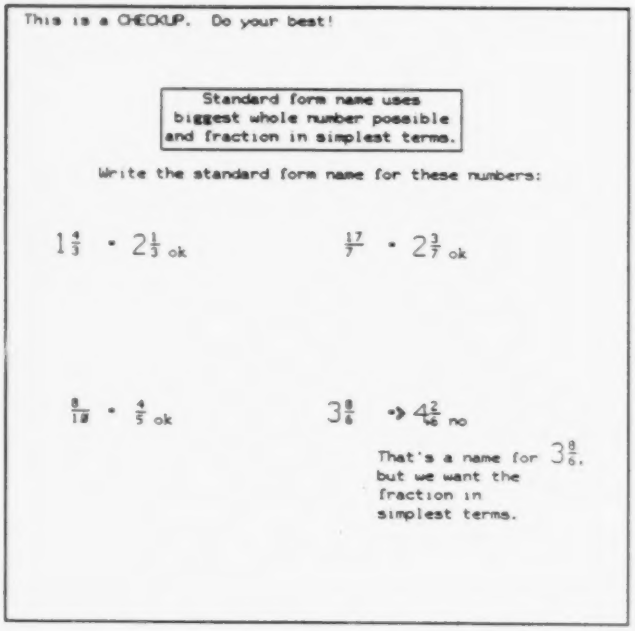

This is a CHECKUP. Do your best!
Standard form name uses biggest whole number possible and fraction in simplest terms.
Write the standard form name for these numbers:
$1\frac{4}{3}$ • $2\frac{1}{3}$ ok
$\frac{17}{7}$ • $2\frac{3}{7}$ ok
$\frac{8}{10}$ • $\frac{4}{5}$ ok
$3\frac{9}{6}$ → $4\frac{2}{6}$ no
That's a name for $3\frac{9}{6}$, but we want the fraction in simplest terms.

**Figure 1**

**Lesson** **Checkup: Add and Subtract Mixed Numbers**

(*SYS 4 catalog name*: **Checkup: Addition and Subtraction**)

**File** *pad2 (SYS 4 file name: pad2.4)*

**Authors** Sharon Dugdale
David Kibbey
Barry Cohen
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To review addition and subtraction of fractions and mixed numbers with like denominators.

**Description** To receive credit, students must correctly answer five problems on the first try, without help, and make fewer than three errors on standard form. They are allowed to press HELP one time and still pass the checkup, if the other criteria for passing are met.

**Interaction**

- Students add fractions and mixed numbers, and then must convert their answers to standard form (see Figure 1).
- HELP is available after two wrong answers on the same problem. Pizzas representing the numbers are shown, and the problem is broken down into smaller steps. Students are then led to the correct answer.
- Students must meet mastery criteria to receive credit for the lesson.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (self-paced)

**Associated Lessons** *Addition and Subtraction with Simplifying and Borrowing*
*Subtraction with Borrowing*

This is a CHECKUP. Do your best!
5/8
+ 5/8
10/8 -> 1 2/8
Good, but put
the fraction part
in simplest terms.

**Figure 1**

# Unit 12

## Adding and Subtracting Fractions with Unlike Denominators

**Objective** To add and subtract fractions with unlike denominators.

**Description** Students are taught to work addition and subtraction problems with unlike denominators.

When students fail a *pretest* and are given the standard assignment, they will see all but the last four lessons in the list below (the bracketed lessons will not be seen).

However, the standard assignment after failure of a *posttest* is the series of lessons marked with an asterisk. (This is the only unit with an automatic change of assignment upon failure of a *posttest*.)

**Lessons**

- Paint Addition: The Computer Paints
  Addition by Using an Easier Problem 1
  Addition by Using an Easier Problem 2
  Paint Addition: You Paint
  High Wire•
  Addition and Subtraction Drill [Unlike Denominators]•
  Checkup: Paint Addition
  Checkup: Unlike Denominators
  {Equivalent Fractions Generalized}•
  {Addition with Equivalence Sets}•
  {Add and Subtract with Equivalence Sets}•
  {Strategies for Listing Equivalence Sets}•

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | |
|---|---|
| 1A, 1B, 1C | More Adding and Subtracting Fractions; Finding LCD |
| 3A, 3B, 3C | Finding LCD When LCD Is Not One of the Fractions to Be Added or Subtracted |
| 5A, 5B, 5C | Adding and Subtracting Mixed Numbers with Unlike Denominators |

---

{ } Lesson will not be assigned after failure of a pretest unless the instructor chooses to do so. (See Preface for explanation.)

• Lessons assigned on failure of a posttest. (See Unit Description above.)

**Lesson** **Paint Addition: The Computer Paints**

**File** *recadnolib (SYS 4 file name: recadnolib.1)*

**Authors** Sharon Dugdale
David Kibbey
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To add fractions with unlike denominators.

**Description** Students identify fractional parts of a box that the computer has painted in two different paints. Students add the two fractions to tell what portion of the box is painted altogether.

**Interaction**

- The computer shows students a box with a portion painted (see Figure 1).
- Students are asked what fraction of the box is painted. They can draw and erase lines on the box to help them identify the fractions.
- The computer leads students through the steps required to add two painted portions together, showing a number sentence after students have determined all the information needed (see Figure 2).
- If three wrong answers are given, students receive help in determining the right answer (see Figure 3).
- Students must reach mastery level.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons** *Cut and Paint*
*Checkup: Cut and Paint*
*Paint Addition: You Paint*
*Checkup: Paint Addition*

**Special Note** Touch capability required

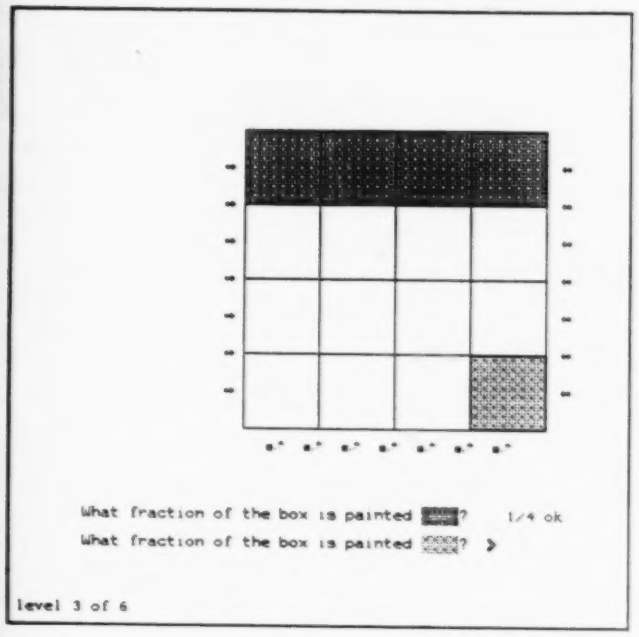



Figure 1

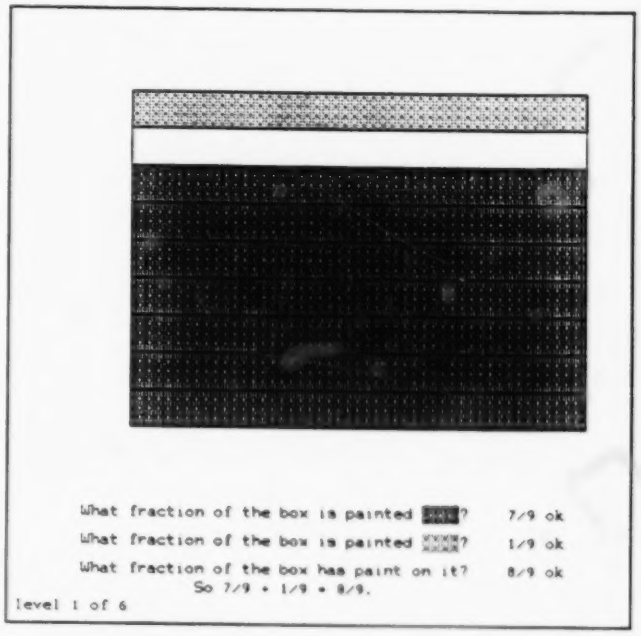

What fraction of the box is painted ?  7/9 ok
What fraction of the box is painted ?  1/9 ok
What fraction of the box has paint on it?  8/9 ok
So 7/9 + 1/9 + 8/9.
level 1 of 6

Figure 2

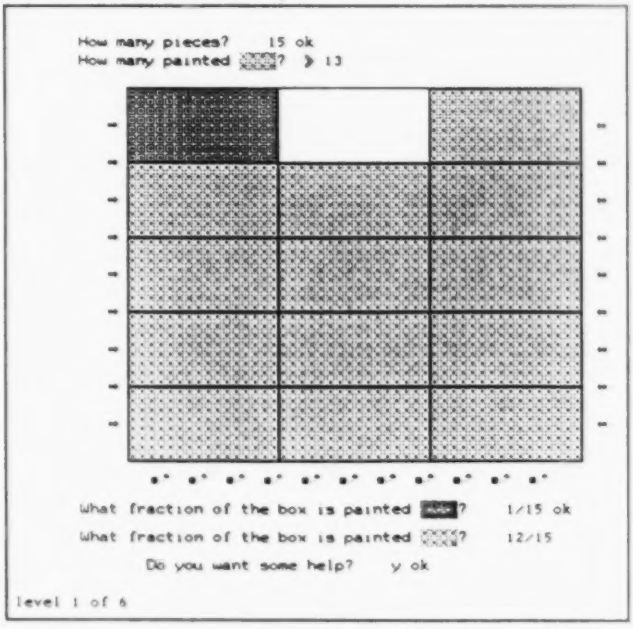

How many pieces?  15 ok
How many painted ?  › 13
What fraction of the box is painted ?  1/15 ok
What fraction of the box is painted ?  12/15
Do you want some help?  y ok
level 1 of 6

Figure 3

**Lesson** **Addition by Using an Easier Problem 1**

**File** *hsrev (SYS 4 file name: hsrev,1)*

**Authors** Sharon Dugdale
David Kibbey
Helen Lueng

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To develop a strategy for adding and subtracting fractions with unlike denominators.

**Description** Students are presented with a problem solving strategy: make an easy problem out of a difficult one by renaming the fractions to have common denominators.

**Interaction**

- Students are given an addition or subtraction problem involving fractions or mixed numbers with unlike denominators (see Figure 1).
- Various characters lead students to discover that renaming the fractions to have common denominators will make the problem easier (see Figure 2).
- Special messages are given for common errors (see Figure 3).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (self-paced)

**Associated Lesson** *Addition by Using an Easier Problem 2*

**Figure 1**

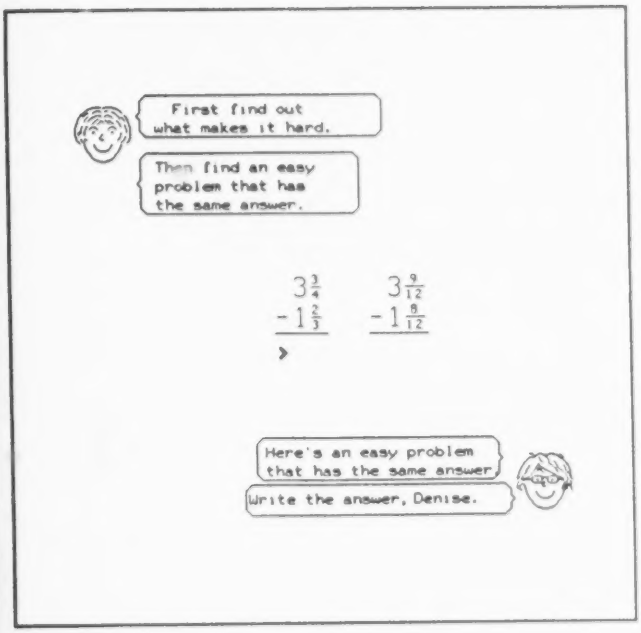

First find out what makes it hard.
Then find an easy problem that has the same answer.
$3\frac{3}{4}$ $3\frac{9}{12}$
$-1\frac{2}{3}$ $-1\frac{8}{12}$
Here's an easy problem that has the same answer.
Write the answer, Denise.

**Figure 2**

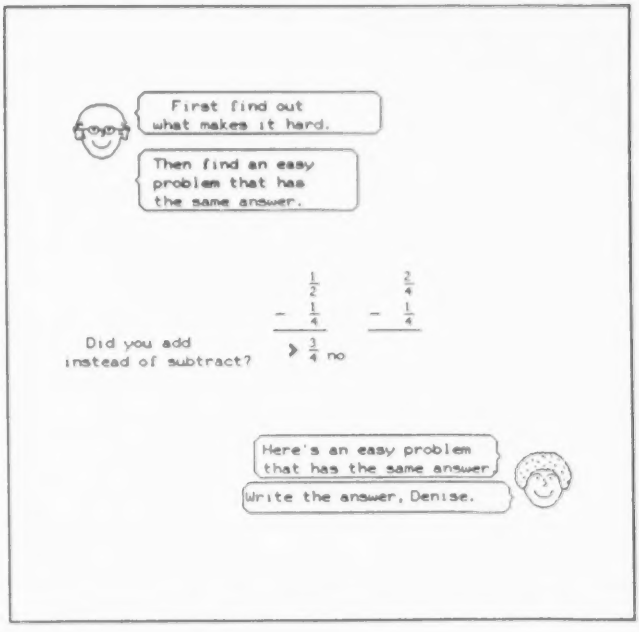

First find out what makes it hard.
Then find an easy problem that has the same answer.
$\frac{1}{2}$ $\frac{2}{4}$
$-\frac{1}{4}$ $-\frac{1}{4}$
Did you add instead of subtract?
$\frac{3}{4}$ no
Here's an easy problem that has the same answer.
Write the answer, Denise.

**Figure 3**

**Lesson** **Addition by Using an Easier Problem 2**

**File** *hs4p (SYS 4 file name: hs4p.4)*

**Authors** Sharon Dugdale
David Kibbey
Helen Lueng

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To add and subtract fractions with unlike denominators.

**Description** Students practice adding and subtracting fractions with unlike denominators.

**Interaction**

- At lower levels, the computer suggests common denominators (see Figure 1).
- At higher levels, students solve problems when neither original denominator can be used as the common denominator.
- The highest level encourages students to find common denominators themselves, providing feedback only when necessary (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lesson** *Addition by Using an Easier Problem 1*

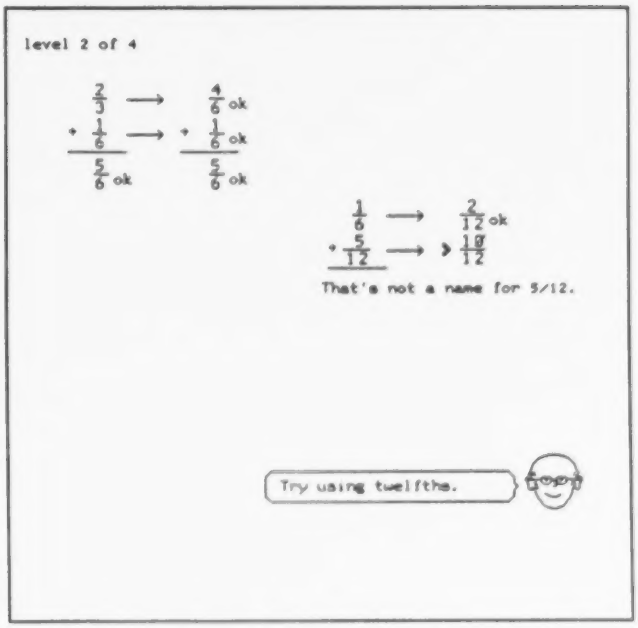



**Figure 1**

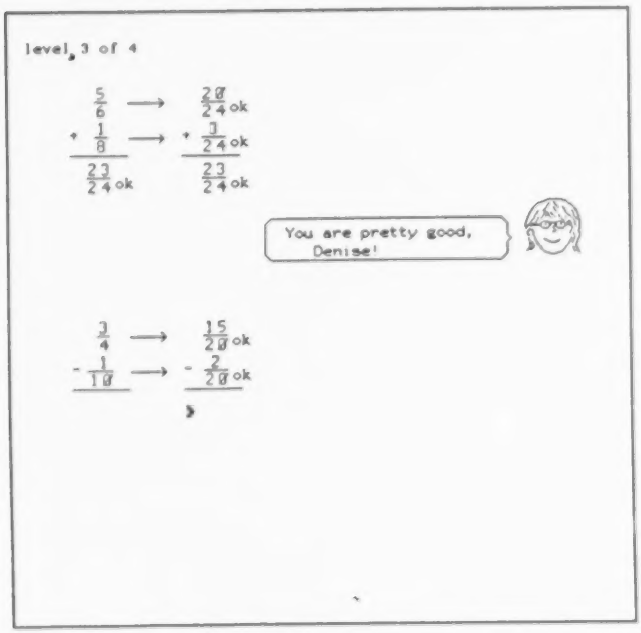



**Figure 2**

**Lesson** **Paint Addition: You Paint**

**File** *recadnolib (SYS 4 file name: recadnolib,4)*

**Authors** Sharon Dugdale
David Kibbey
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To add fractions with unlike denominators.

**Description** Students paint fractions of a box with two colors, describing the fractions given. Students add the two fractions together to tell what portion of the box is painted.

**Interaction**

- Students see a blank box and are asked to paint one fraction of it with one pattern and another fraction of it with a different pattern. They must draw their own lines to make boundaries for the paint (see Figure 1).
- If students answer incorrectly, they are led to the right answer (see Figure 2).
- At Level 3 and beyond, lines must be drawn in both directions in order to have enough divisions to solve the problems.
- The level of problems adjusts to student performance.
- Students work until they reach mastery level.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons** *Cut and Paint*
*Checkup: Cut and Paint*
*Paint Addition: The Computer Paints*
*Checkup: Paint Addition*

**Special Note** Touch capability required

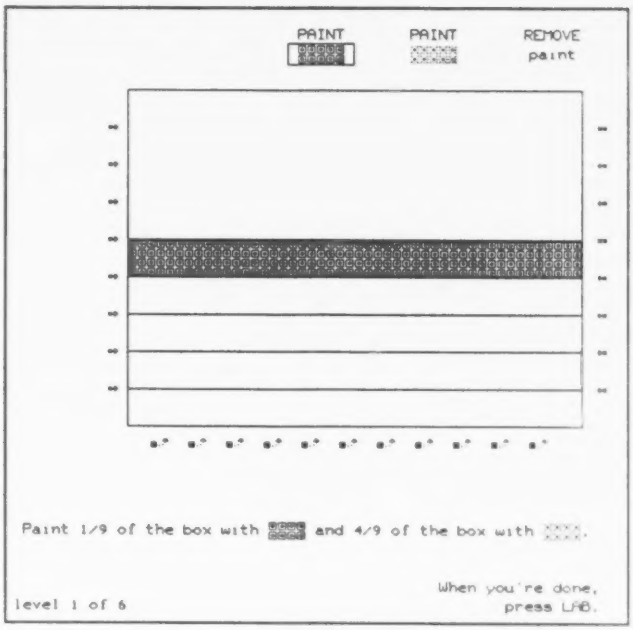



**Figure 1**

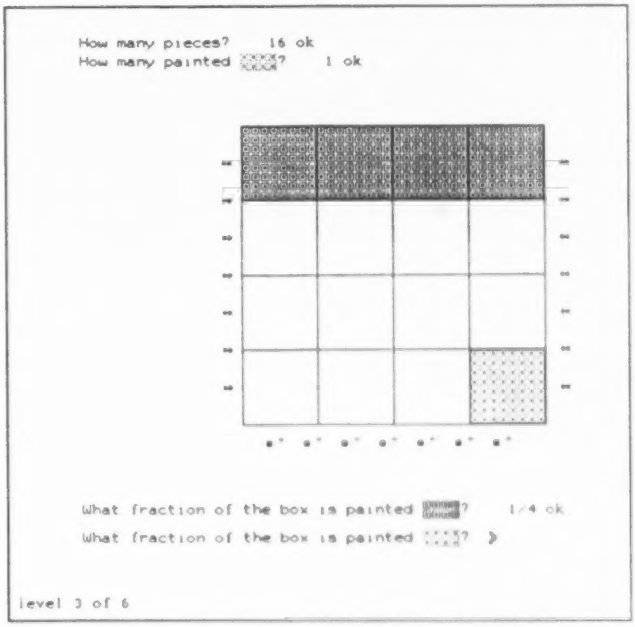

How many pieces? 16 ok
How many painted ? 1 ok
What fraction of the box is painted ? 1/4 ok
What fraction of the box is painted ?
level 3 of 6

**Figure 2**

**Lesson** **High Wire**

**File** *monkey (SYS 4 file name: monkey.1)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To add fractions, with unlike denominators, on the number line.

**Description** This animated lesson illustrates common denominators when adding fractions on the number line. A monkey makes two swings down the number line, making an addition problem. The students' job is to catch him to tickle him with a feather. If the monkey is caught, he falls to the ground laughing.

**Interaction**

- Students watch a monkey swing along a number line. They are told the length of each swing in fractional notation.
- Students then write a fraction or mixed number representing how far along the number line the feather should travel in order to find the monkey and tickle him (see Figures 1 and 2).
- At first, the fractions are simple. Later, students must do more difficult addition by finding more difficult common denominators.
- If students answer incorrectly, the monkey goes free.
- Level of difficulty adjusts to student performance.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Special Note** Touch capability required

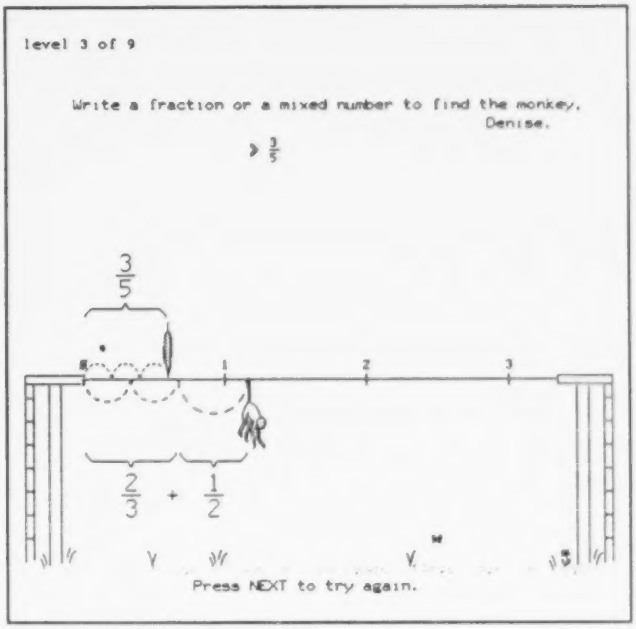



**Figure 1**

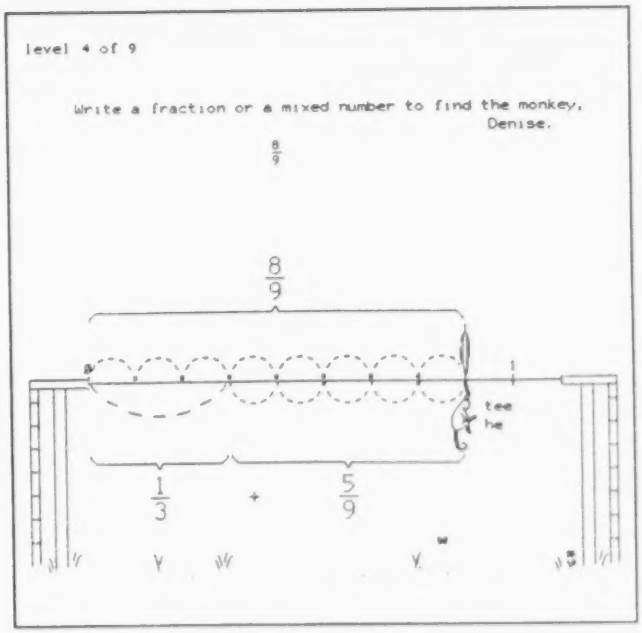



**Figure 2**

**Lesson** **Addition and Subtraction Drill [Unlike Denominators]**

(*SYS 4 catalog name*: **Add and Subtract Unlike Denominators**)

**File** *tryad (SYS 4 file name: tryad,3)*

**Authors** Sharon Dugdale
David Kibbey
Tom Layman
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To add and subtract fractions and mixed numbers with unlike denominators.

**Description** Students work addition and subtraction problems with unlike denominators. While any correct equivalent is acknowledged, answers must end up in simplest terms.

**Interaction**

- Students see four addition or subtraction problems per page.
- Appropriate messages are given for common errors (see Figure 1).
- Help is provided for trouble renaming, borrowing, or putting answers in standard form (see Figure 2).
- Problems are presented horizontally as well as vertically.
- Higher level problems require carrying and borrowing.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 30 minutes (self-paced)

**Associated Lessons** *Checkup: Unlike Denominators*
*Addition and Subtraction Drill [Like Denominators]*

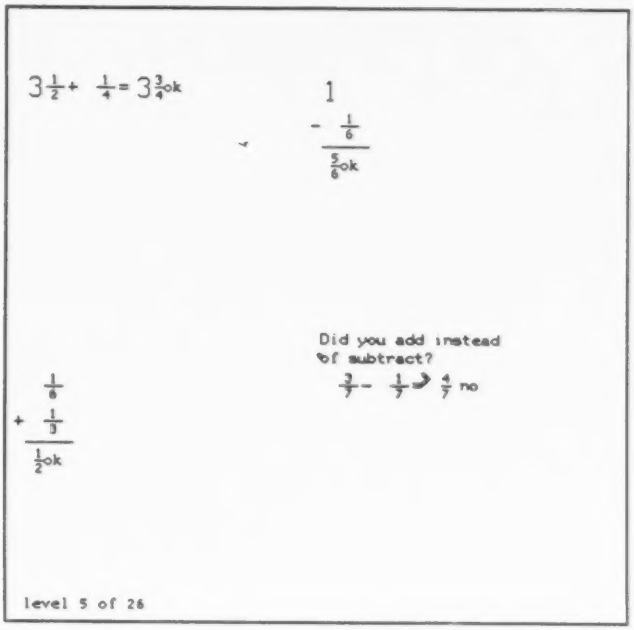



**Figure 1**

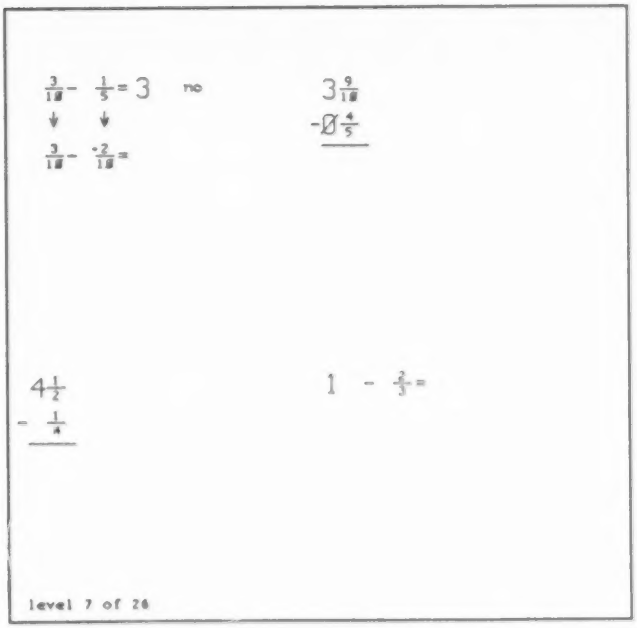



**Figure 2**

**Lesson** ## Checkup: Paint Addition

**File** *recadnolib (SYS 4 file name: recadnolib,3)*

**Authors** Sharon Dugdale
David Kibbey
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To add fractions with unlike denominators.

**Description** This lesson reviews the concepts in *Paint Addition: The Computer Paints.* Students identify fractional parts of a box that have been painted in two different paints. Students add the two fractions to tell what portion of the box is painted. Problems are similar to those in the higher levels of the earlier lesson.

**Interaction**

- Students tell what fraction of the box has been painted (see Figure 1).
- Problems require students to draw extra lines in order to solve the problems (see Figure 2).
- Students work until they achieve mastery level.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 5 minutes (self-paced)

**Associated Lessons**
*Cut and Paint*
*Checkup: Cut and Paint*
*Paint Addition: The Computer Paints*
*Paint Addition: You Paint*

**Special Note** Touch capability required

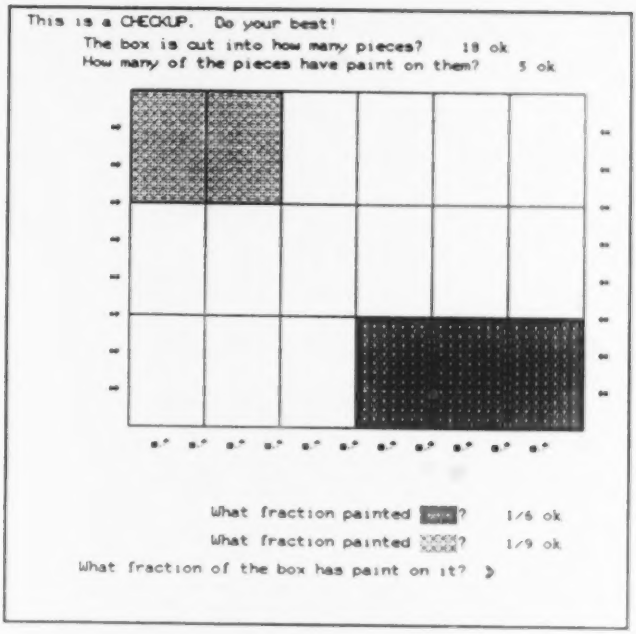

This is a CHECKUP. Do your best!
The box is cut into how many pieces? 18 ok
How many of the pieces have paint on them? 5 ok
What fraction painted ? 1/6 ok
What fraction painted ? 1/9 ok
What fraction of the box has paint on it? >

**Figure 1**

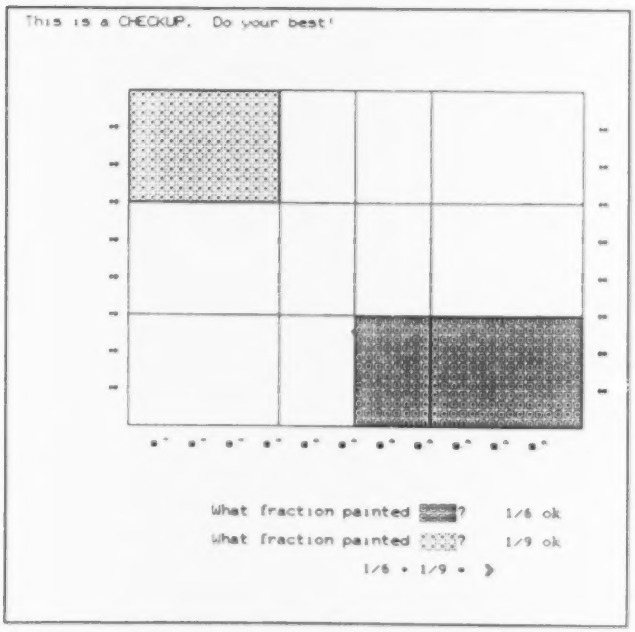



**Figure 2**

**Lesson** ## Checkup: Unlike Denominators

**File** *strad (SYS 4 file name: strad.3)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To add and subtract fractions and mixed numbers with unlike denominators.

**Description** Students practice adding and subtracting fractions and mixed numbers with the option of listing equivalence sets for the fractions to help them find common denominators.

**Interaction**

- Students are given two numbers to add or subtract. Finding a common denominator is required for the solution (see Figure 1).
- Students can choose to create equivalence sets to help find a common denominator (see Figure 2).
- Students must work until they reach mastery level.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Addition and Subtraction Drill [Unlike Denominators]*
*Strategies for Listing Equivalence Sets*

**Special Note** Touch capability required

**Figure 1**

This is a CHECKUP. Do your best!
$3\frac{3}{7}$
$+ 3\frac{1}{2}$
$\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21},$
$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14},$
> 6
Write the answer.
Or press HELP to write more names.

**Figure 2**

**Lesson** **Equivalent Fractions Generalized**

**File** *egen (SYS 4 file name: egen.1)*

**Authors** Sharon Dugdale
David Kibbey
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To systematically find fractions equivalent to a given fraction.

**Description** Through screen dialog and fill-in-the-blank practice, students find equivalence sets for given fractions.

**Interaction**

- Students see a box with a portion painted. They are asked to think of other names for the same portion (see Figure 1).
- Later, they are asked to keep the names in order (see Figure 2).
- At higher levels, they are creating straight equivalence sets.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lessons** *Addition with Equivalence Sets*
*Add and Subtract with Equivalence Sets*
*Strategies for Listing Equivalence Sets*

**Special Note** This lesson is not part of the initial standard assignment. If students do not receive credit on the first posttest, this lesson (and the three following) will be assigned. This alternative assignment presents a different approach to calculations involving unlike denominators.

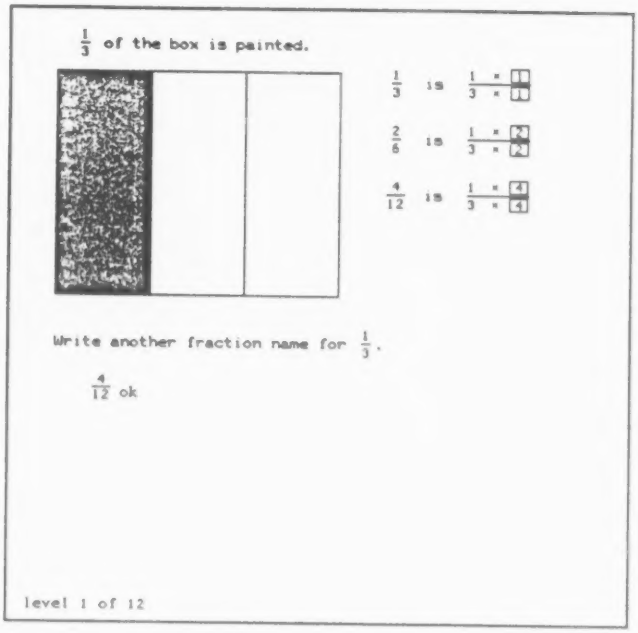



Figure 1

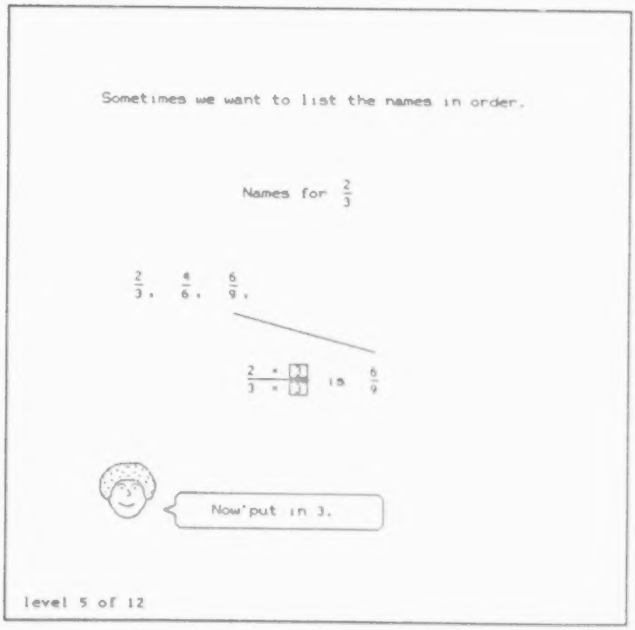



Figure 2

**Lesson** **Addition with Equivalence Sets**

**File** *slad (SYS 4 file name: slad.1)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To find common denominators.

**Description** Students learn to list equivalence sets to find common denominators in order to solve addition problems involving fractions with unlike denominators.

**Interaction**

- Students are led through the process of how to select a common denominator (see Figures 1 and 2).
- Students list equivalence sets and analyze them to find a common denominator (see Figure 3).
- Students then solve the addition problem.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Equivalent Fractions Generalized*
*Add and Subtract with Equivalence Sets*
*Strategies for Listing Equivalence Sets*

**Special Notes**

- Touch capability required
- This lesson is not part of the initial standard assignment. If students do not receive credit on the first posttest, this lesson (along with the associated lessons) will be assigned.

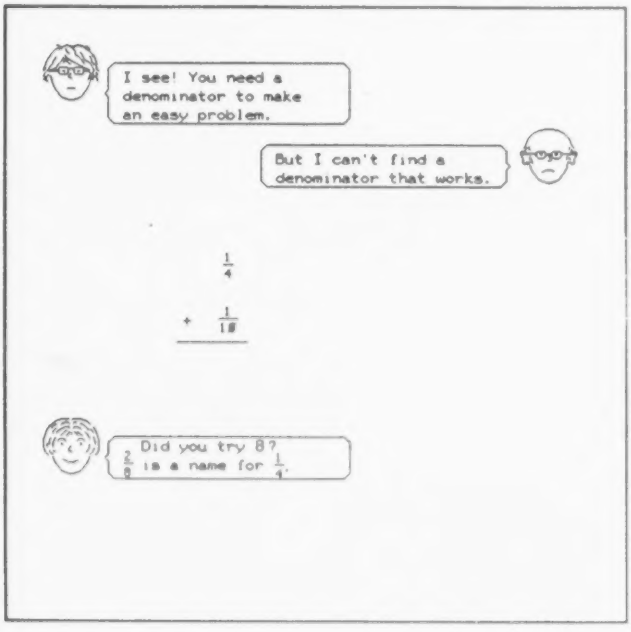

I see! You need a denominator to make an easy problem.
But I can't find a denominator that works.
$\frac{1}{4}$
$+ \frac{1}{10}$
Did you try 8? $\frac{2}{8}$ is a name for $\frac{1}{4}$.

Figure 1

I have an idea! Let's write down all the names!
Ok! Maybe we'll find some that work!
$\frac{1}{4}$ $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \cdots\right\}$
$+ \frac{1}{10}$ $\left\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \cdots\right\}$
Now write the equivalence set for $\frac{1}{10}$.

Figure 2

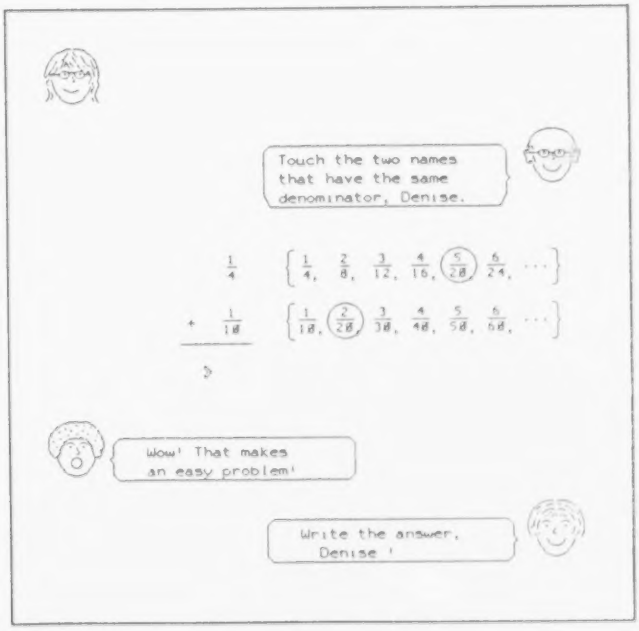

Touch the two names that have the same denominator, Denise.
$\frac{1}{4}$ $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \cdots\right\}$
$+ \frac{1}{10}$ $\left\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \frac{5}{50}, \frac{6}{60}, \cdots\right\}$
Wow! That makes an easy problem!
Write the answer, Denise!

Figure 3

**Lesson** ## Add and Subtract with Equivalence Sets

(*SYS 4 catalog name*: **Adding/Subtracting with Equivalence Sets**)

**File** *slad2 (SYS 4 file name: slad2.1)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**
- To find common denominators.
- To add and subtract fractions and mixed numbers with unlike denominators.

**Description** Students learn to systematically list equivalence sets to find common denominators. They then add or subtract and check for simplest terms.

**Interaction**
- Students list equivalence sets and then analyze them to find a common denominator (see Figure 1).
- Special messages are provided for common errors (see Figure 2).
- Higher level problems involve carrying and borrowing.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons**
*Equivalent Fractions Generalized*
*Addition with Equivalence Sets*
*Strategies for Listing Equivalence Sets*

**Special Notes**
- Touch capability required
- This lesson is not part of the initial standard assignment. If students do not receive credit on the first posttest, this lesson (and its associated lessons) will be assigned.

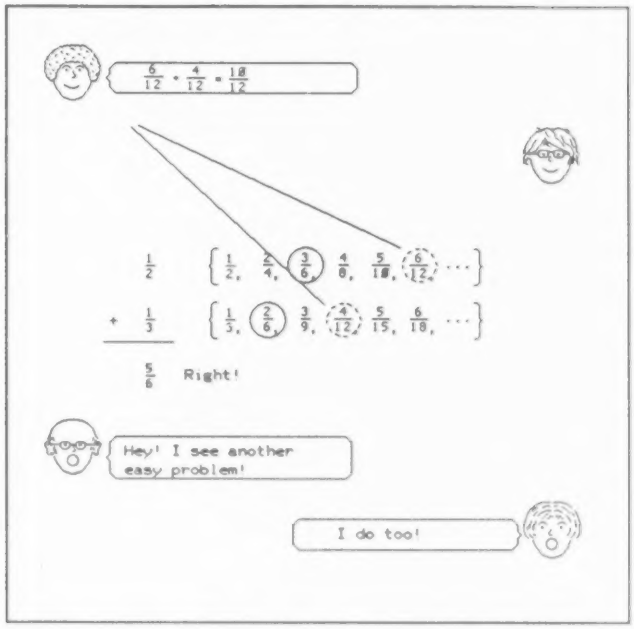



**Figure 1**

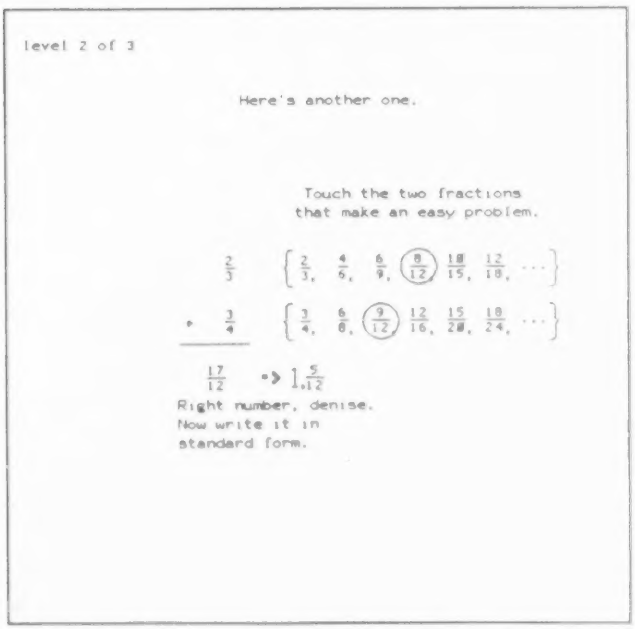

level 2 of 3
Here's another one.
Touch the two fractions that make an easy problem.
2/3 {2/3, 4/6, 6/9, 8/12, 10/15, 12/18, ...}
+ 3/4 {3/4, 6/8, 9/12, 12/16, 15/20, 18/24, ...}
17/12 -> 1 5/12
Right number, denise.
Now write it in standard form.

**Figure 2**

## Lesson Strategies for Listing Equivalence Sets

**File** *strad (SYS 4 file name: strad.1)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To find common denominators.
- To add and subtract fractions and mixed numbers.

**Description** Students learn to systematically list equivalence sets to find common denominators. They then add or subtract and check for simplest terms.

**Interaction**

- Students create equivalence sets and then analyze them to find a common denominator (see Figure 1).
- Special messages are provided for common errors (see Figures 2 and 3).
- Students must learn to do the problems without listing the equivalence sets in order to receive credit for the lesson (see Figure 4).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lessons**
*Equivalent Fractions Generalized*
*Addition with Equivalence Sets*
*Add and Subtract with Equivalence Sets*
*Checkup: Unlike Denominators*

**Special Notes**

- Touch capability required
- This lesson is not part of the initial standard assignment. If students do not receive credit on the first posttest, this lesson (along with the associated lessons) will be assigned.

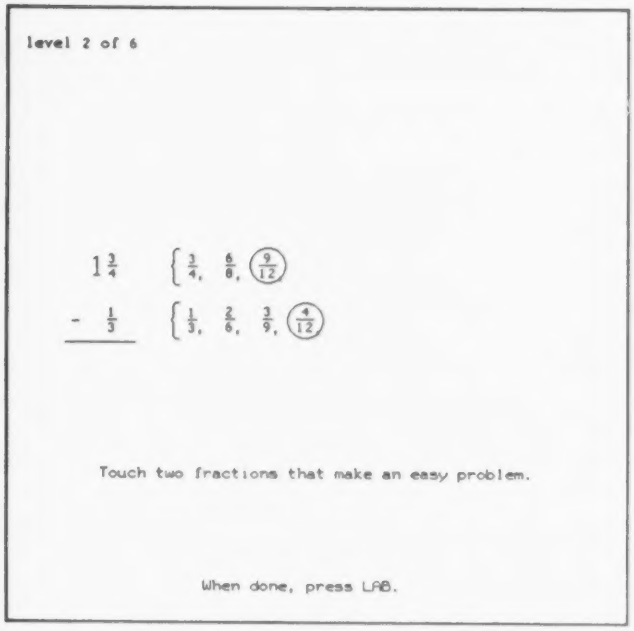



Figure 1

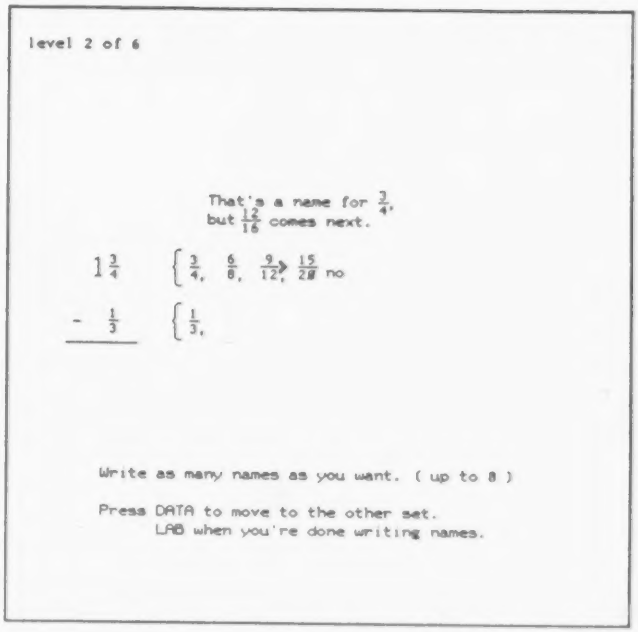

level 2 of 6
That's a name for $\frac{3}{4}$, but $\frac{12}{16}$ comes next.
$1\frac{3}{4}$
$-\ \frac{1}{3}$
$\frac{3}{4}$, $\frac{6}{8}$, $\frac{9}{12}$, $\frac{15}{20}$ no
$\frac{1}{3}$,
Write as many names as you want. ( up to 8 )
Press DATA to move to the other set.
LAB when you're done writing names.

Figure 2

Figure 3

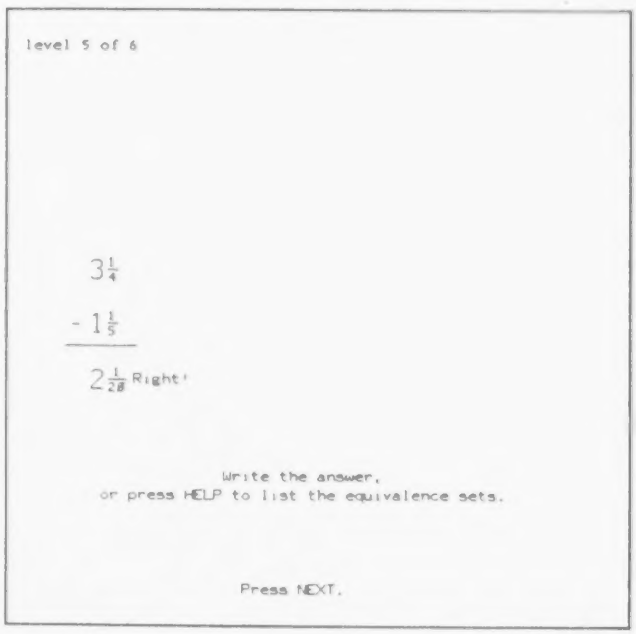



Figure 4

# Unit 13

## Multiplying and Dividing Fractions

**Objective** To multiply and divide fractions and mixed numbers.

**Description** Students are taught to work multiplication and division problems involving fractions.

**Lessons**

- Introduction to Multiplying Mixed Numbers
  Multiplication Practice with Pictures for Help
  Multiplication of Fractions
  Strategies for Multiplying Mixed Numbers
  Mixed Number Multiplication Practice
  Practice with Fractions

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | Multiplying Fractions |
| 3A, | 3B, | 3C | Multiplying by Whole Numbers |
| 5A, | 5B, | 5C | Cancelling in Multiplication |
| 7A, | 7B, | 7C | Multiplying Mixed Numbers |
| 10A, | 10B, | 10C | Dividing Fractions |
| 12A, | 12B, | 12C | Dividing with Fractions and Whole Numbers |
| 14A, | 14B, | 14C | Cancelling in Division |
| 16A, | 16B, | 16C | Dividing by Mixed Numbers |

**Lesson** ## Introduction to Multiplying Mixed Numbers

(*SYS 4 catalog name*: **Multiplying Mixed Numbers, Intro.**)

**File** *fmint*

**Authors** Sharon Dugdale
David Kibbey
Marilyn Bereiter

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To multiply whole numbers, fractions, and mixed numbers using an array of boxes to help visualize the process.

**Description** This lesson introduces the multiplication of mixed numbers, using an array of boxes to help students count the parts.

The lesson begins by using only whole numbers. Once students understand how the array of boxes works, an extra half box is added. Students determine how many boxes there are by putting half boxes together to make whole ones. Later problems ask students to write multiplication number sentences describing the arrays.

**Interaction**

- Students are shown an array of boxes and led through the steps creating a number sentence to describe the array (see Figure 1).
- Students then write number sentences on their own describing the arrays (see Figure 2).
- Lower levels use whole numbers.
- Higher levels use mixed numbers in both dimensions.
- Students try until they answer correctly.
- Students must meet criteria at each level to receive credit.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lesson** *Multiplication Practice with Pictures for Help*

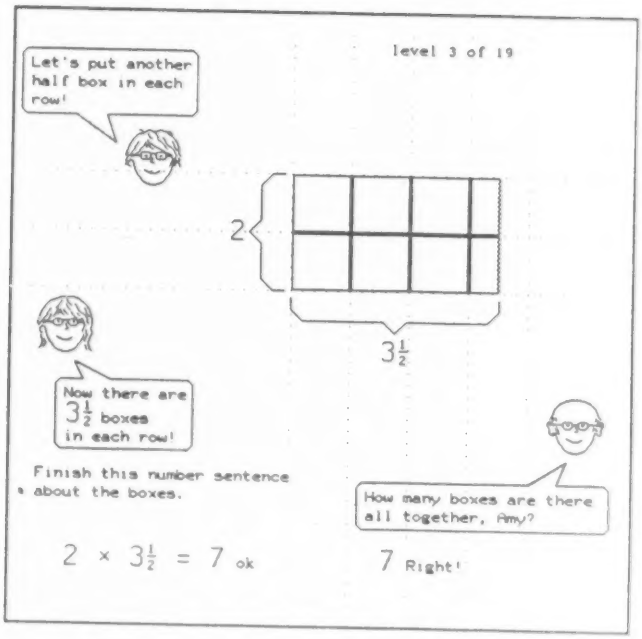

level 3 of 19
Let's put another half box in each row!
2
3 1/2
Now there are 3 1/2 boxes in each row!
Finish this number sentence about the boxes.
2 × 3 1/2 = 7 ok
How many boxes are there all together, Amy?
7 Right!

**Figure 1**

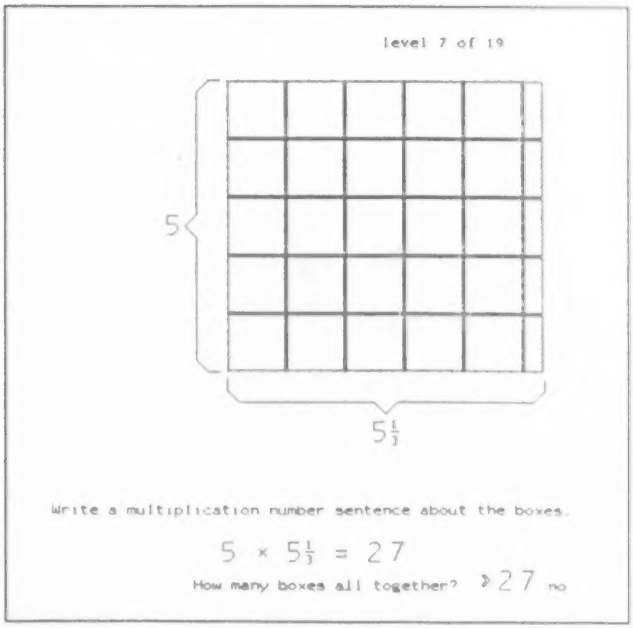



**Figure 2**

**Lesson** **Multiplication Practice with Pictures for Help**

(*SYS 4 catalog name*: **Multiplying with Pictures for Help**)

**File** *fmpix*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice multiplying fractions and whole numbers using arrays of boxes.

**Description** This lesson builds on skills developed in the previous lesson, *Introduction to Multiplying Mixed Numbers*. Students begin to work problems without pictures, although pictures are available if they ask for them.

**Interaction**

- Students are presented with a problem multiplying a whole number and either a fraction or a mixed number (see Figure 1).
- HELP for a picture of the problem, in the form of an array of boxes, is always available (see Figure 2).
- Standard form is required for credit.
- Students must meet criteria at each level for credit.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (self-paced)

**Associated Lesson** *Introduction to Multiplying Mixed Numbers*

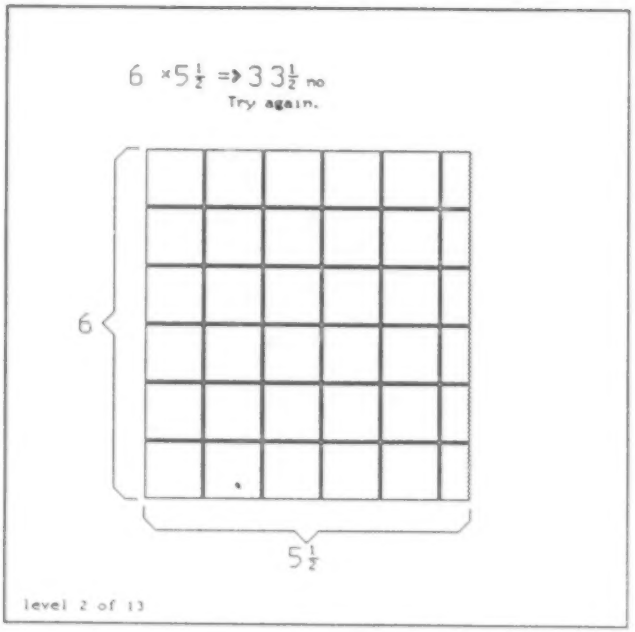



**Figure 1**

**Figure 2**

**Lesson** **Multiplication of Fractions**

**File** *fmult*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To multiply fractions.

**Description** Students learn multiplication of fractions by finding the area covered in an array of boxes.

All of the problems involve fractions rather than mixed numbers, so students must think of fractions of boxes and fractions of rows. The finished problems are saved at the bottom of the screen. After six problems are completed, the program shows a pattern in the problems and suggests a shortcut for finding answers (multiplying numerators and denominators). Students then test the pattern to verify it.

The program offers an optional explanation of why the shortcut works, involving the same pictorial approach.

**Interaction**

- Students are given a problem to solve and are shown how to picture it as part of an array of boxes (see Figure 1).
- After several problems are done, a pattern is pointed out, indicating that multiplication of the two numerators yields the new numerator, and multiplication of the denominators yields the new denominator (see Figure 2).
- Students then practice using this method for multiplying fractions.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (self-paced)

**Associated Lesson** *Strategies for Multiplying Mixed Numbers*

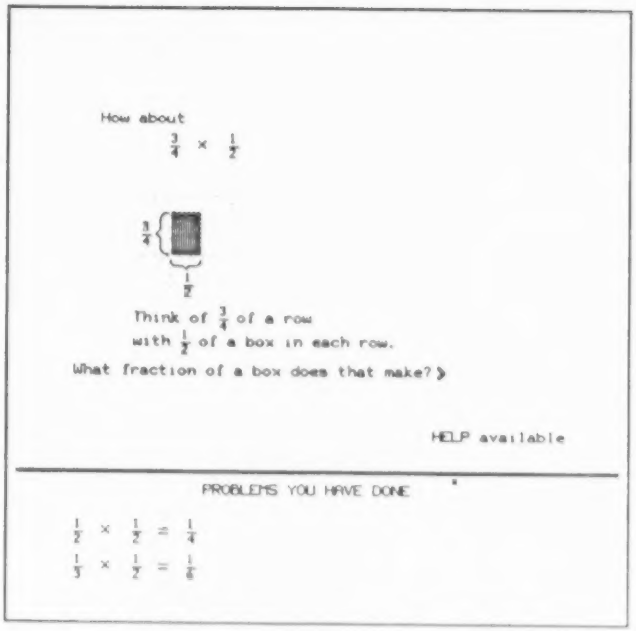

How about
$\frac{3}{4} \times \frac{1}{2}$
$\frac{3}{4}$
$\frac{1}{2}$
Think of $\frac{3}{4}$ of a row
with $\frac{1}{2}$ of a box in each row.
What fraction of a box does that make?
HELP available
PROBLEMS YOU HAVE DONE
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

**Figure 1**

Look at the problems Amy has done!
You can multiply the top parts to get the top part of the answer.
I see! And you can multiply the bottom parts to get the bottom part of the answer!
1 × 1 = 1 ok
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
2 × 2 = 4 ok
It works for this one! NEXT for another.
Let's make sure the short cut works for all of them.
PROBLEMS YOU HAVE DONE
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$
$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$
$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$
$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

**Figure 2**

**Lesson** **Strategies for Multiplying Mixed Numbers**

(*SYS 4 catalog name*: **Strategies to Multiply Mixed Numbers**)

**File** *fmstrat*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To multiply mixed numbers.

**Description** This lesson demonstrates that there are many ways to multiply mixed numbers. Students examine problems and choose an appropriate method to solve each one.

The methods include repeated addition; making an array of boxes; multiplying the whole numbers first, then, the whole number and fraction; cancelling; converting mixed number to improper fraction; and multiplying.

Students practice each method as it is presented.

Each practice set reinforces previous types of problems as well as working on the current type.

**Interaction**

- Students are led through several ways to multiply mixed numbers (see Figures 1 and 2).
- After each method is introduced, students practice it. (Previously studied methods are also reviewed.) (See Figure 3.)
- HELP is available during practice.
- Students keep trying until they find the right answer.
- Level of difficulty adjusts to student performance.
- While equivalent names are accepted, answers in standard form are required for credit.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 30 minutes (self-paced)

**Associated Lesson** *Multiplication of Fractions*

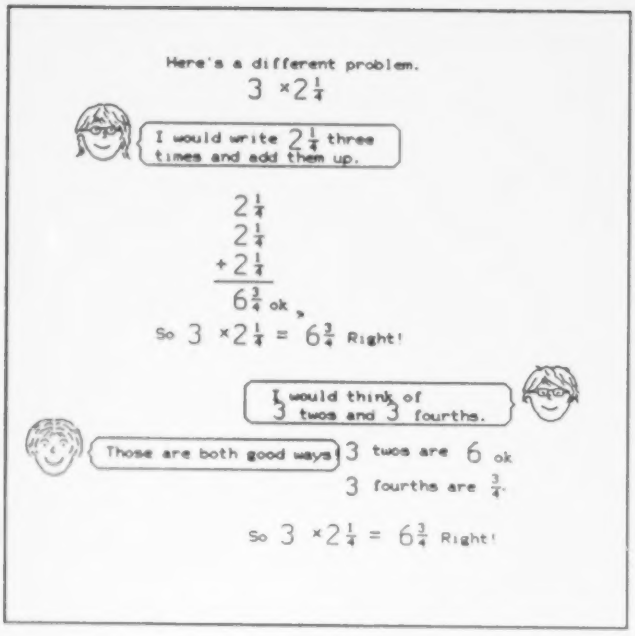

Here's a different problem.
3 × 2 1/4
I would write 2 1/4 three times and add them up.
2 1/4
2 1/4
+ 2 1/4
6 3/4 ok
So 3 × 2 1/4 = 6 3/4 Right!
I would think of 3 twos and 3 fourths.
Those are both good ways!
3 twos are 6 ok
3 fourths are 3/4.
So 3 × 2 1/4 = 6 3/4 Right!

**Figure 1**

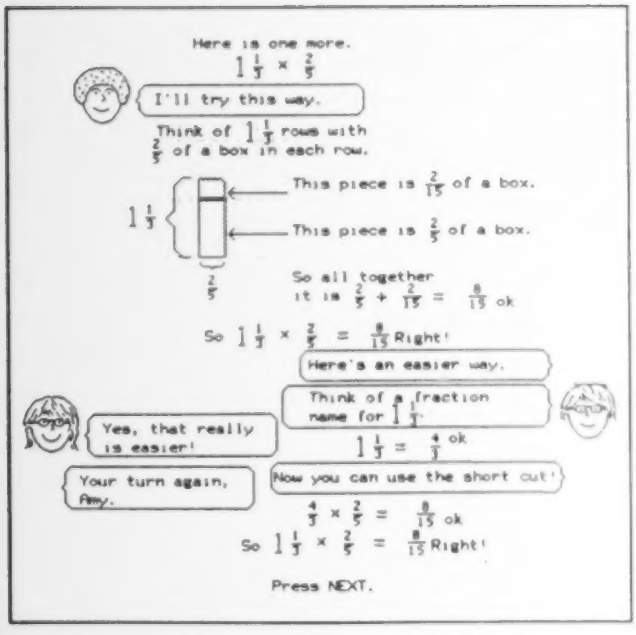

Here is one more.
1 1/3 × 2/5
I'll try this way.
Think of 1 1/3 rows with 2/5 of a box in each row.
This piece is 2/15 of a box.
This piece is 2/5 of a box.
So all together it is 2/5 + 2/15 = 8/15 ok
So 1 1/3 × 2/5 = 8/15 Right!
Here's an easier way.
Think of a fraction name for 1 1/3.
Yes, that really is easier!
1 1/3 = 4/3 ok
Your turn again, Amy.
Now you can use the short cut!
4/3 × 2/5 = 8/15 ok
So 1 1/3 × 2/5 = 8/15 Right!
Press NEXT.

**Figure 2**

level 1 of 34
Think of different ways to find the answer.
Choose a way that is easy for you.
3 1/4 × 4 =
You could think of 3 1/4 fours.
You could think of 3 1/4 rows with 4 boxes in each row.

**Figure 3**

**Lesson** **Mixed Number Multiplication Practice**

**File** *fmdrill*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice multiplying mixed numbers.

**Description** Students practice multiplying mixed numbers. At lower levels, students can ask for a hint suggesting a method for solving the particular problem. At higher levels, HELP is not available.

**Interaction**

- Four problems appear at a time (see Figure 1).
- Hints are related to the type of problem (see Figures 2 and 3). They are verbal rather than graphic, and they assume students know the content covered in the previous lesson, *Strategies for Multiplying Mixed Numbers*.
- Students keep trying until they get each problem right.
- Level of difficulty adjusts to student performance.
- Students must work until they reach mastery level.
- If students choose to do more problems after completing the lesson, they are returned to Level 9. They are allowed to work as long as they like.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 30 minutes (self-paced)

**Associated Lesson** *Strategies for Multiplying Mixed Numbers*

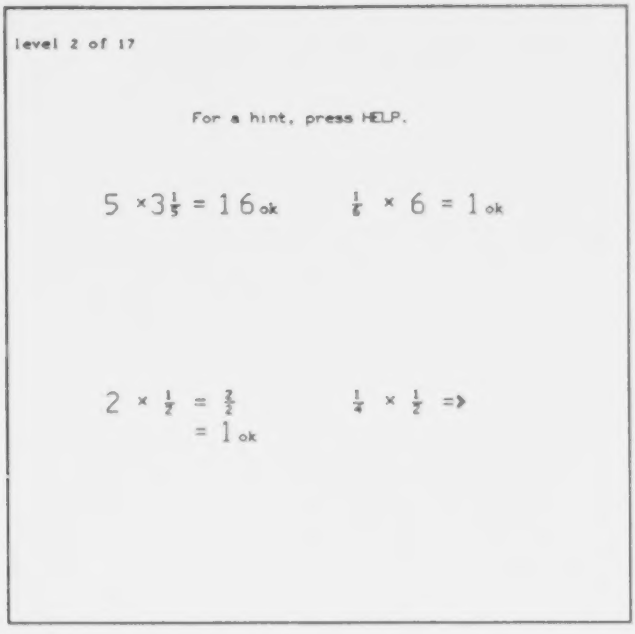



**Figure 1**

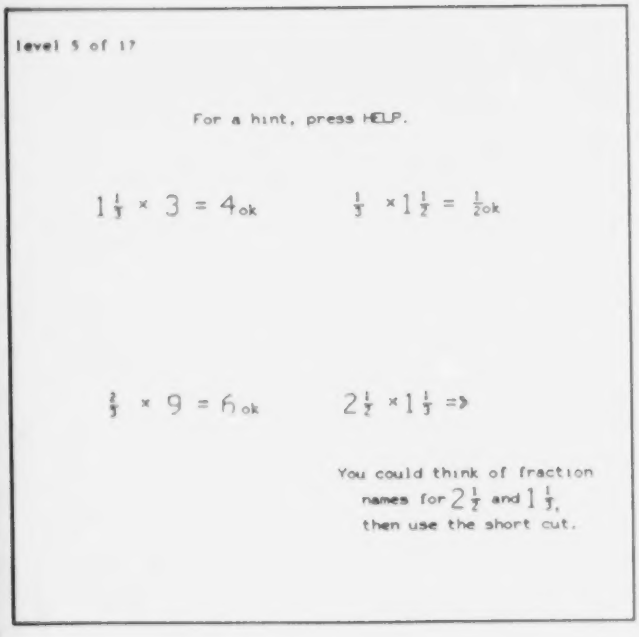

level 5 of 17
For a hint, press HELP.
1 1/3 × 3 = 4 ok
1/3 × 1 1/2 = 1/2 ok
2/3 × 9 = 6 ok
2 1/2 × 1 1/3 =>
You could think of fraction
names for 2 1/2 and 1 1/3,
then use the short cut.

**Figure 2**

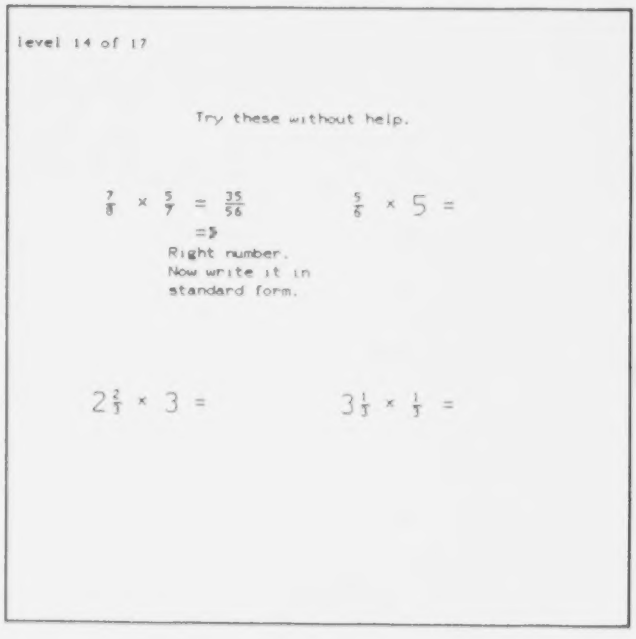



**Figure 3**

**Lesson** ## Practice with Fractions

**File** *fracprac*

**Authors** Keith Bailey
David Lassner

**Author Affiliation** Community College Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To provide practice and a quiz on the four arithmetic operations on fractions.

**Description** This six-part lesson gives students practice in performing the four operations with fractions. It ends with a set of mixed exercises that students must master to complete the lesson. Students can skip directly to the test if they wish.

Each section has three options: instruction on an appropriate rule with a typical problem explained, practice, and a quiz.

**Interaction**

- Students choose which section they wish to study (or they can choose to take the test). (See Figure 1.)
- Once inside a section, students can choose to see the rules for that operation, work practice problems, or take a quiz on that section (see Figure 2).
- While doing practice problems, students can ask to see the rule (see Figure 3) to use or ask for help in stepping through the problem (see Figure 4).
- Students are not required to use standard form.
- Equivalent names are accepted.
- Correct answers are given after three errors on a problem.
- Missed items are not reviewed (unless students choose to do more practice).
- Students earn credit for the lesson upon successful completion of the final quiz.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** One hour (self-paced)



137

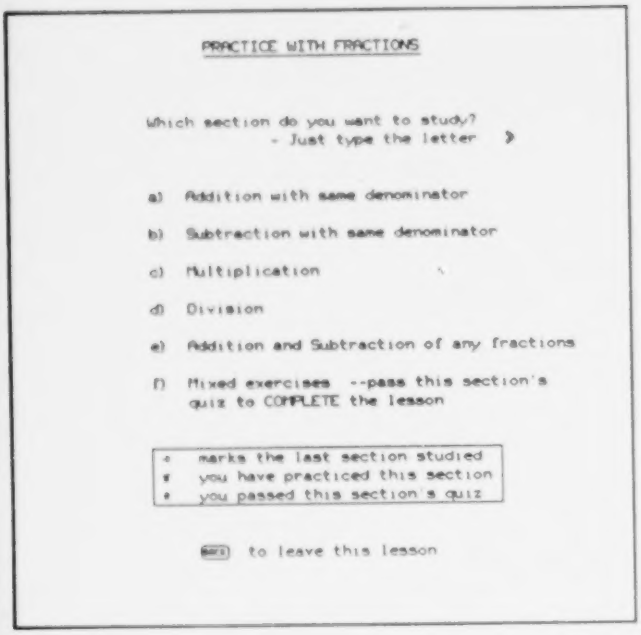

PRACTICE WITH FRACTIONS
Which section do you want to study?
- Just type the letter >
a) Addition with same denominator
b) Subtraction with same denominator
c) Multiplication
d) Division
e) Addition and Subtraction of any fractions
f) Mixed exercises --pass this section's quiz to COMPLETE the lesson
* marks the last section studied
# you have practiced this section
+ you passed this section's quiz
BACK to leave this lesson

**Figure 1**

Multiplication of fractions
NEXT for instruction over this section
DATA to practice this section
LAB for the quiz over this section
BACK to return to the index
BACK will return you to this page

**Figure 2**

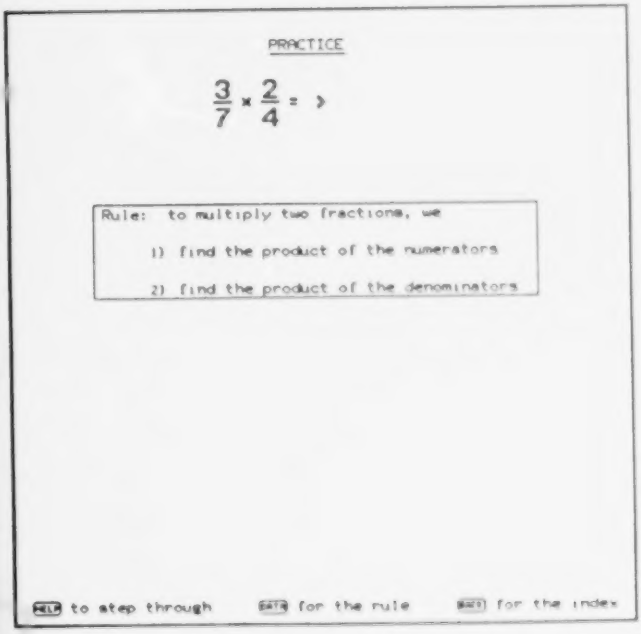

PRACTICE
3/7 x 2/4 = >
Rule: to multiply two fractions, we
1) find the product of the numerators
2) find the product of the denominators
HELP to step through
DATA for the rule
BACK for the index

**Figure 3**

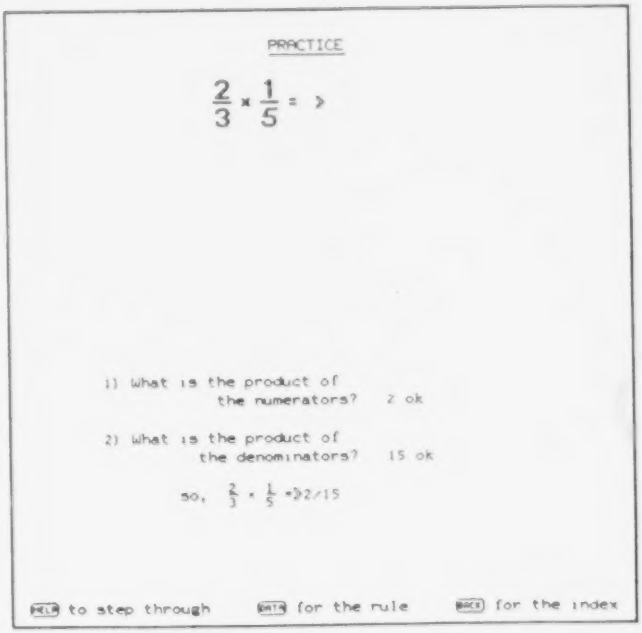

PRACTICE
2/3 x 1/5 = >
1) What is the product of the numerators? 2 ok
2) What is the product of the denominators? 15 ok
so, 2/3 x 1/5 =>2/15
HELP to step through
DATA for the rule
BACK for the index

**Figure 4**

# Unit 14

## Adding and Subtracting Decimals

**Objective** To add and subtract decimals.

**Description** This unit serves as an introduction to decimals. Students are taught to read and write decimals according to conventional methods. They work on lining up decimal points and adding decimal numbers. They also are taught to recognize and differentiate mixed numbers, common fractions, whole numbers, and decimal numbers; to convert between decimals and common fractions; and to determine which of two fractional numbers is larger.

**Lessons**

- {Decimal Darts}

Decimal Skills I: Reading and Writing Decimals
Decimal Skills IIa: Adding Decimals
Decimal Skills IIb: Subtracting Decimals
Decimal Pinball
Decimal and Fraction Conversions
Sort Number Names with Decimals
Decimal Skills V: Comparing Decimals and Fractions

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | | | |
|---|---|---|---|
| 1A, | 1B, | 1C | What is a Decimal? |
| 3A, | 3B, | 3C | Adding Decimals |
| 5A, | 5B, | 5C | Subtracting Decimals |

{ } Lesson will not be assigned unless the instructor chooses to do so. (See Preface for explanation.)

**Lesson** **Decimal Darts**

**File** *ddarts*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**

- To write decimal notation.
- To use decimals to locate points on the number line.

**Description** Students practice locating points on a number line by shooting darts at balloons tied along the number line (see Figure 1). The computer accepts decimals expressed in tenths, hundredths, and thousandths. It does not accept answers expressed as fractions.

**Interaction**

- Students initially review writing decimals to enhance their performance in the game.
- Students can press HELP to have the computer throw a dart. This gives the students a reference point (see Figure 2).
- The level of difficulty adjusts according to students' responses. The balloons get smaller, and the distance between whole numbers gets larger.
- At lower levels, marked points on a number line are whole numbers. At higher levels, marked points are tenths of a number apart, and students must use hundredths or even thousandths to hit a balloon.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 20 minutes (open-ended)

**Special Note** This lesson will not be assigned unless the instructor chooses to do so.

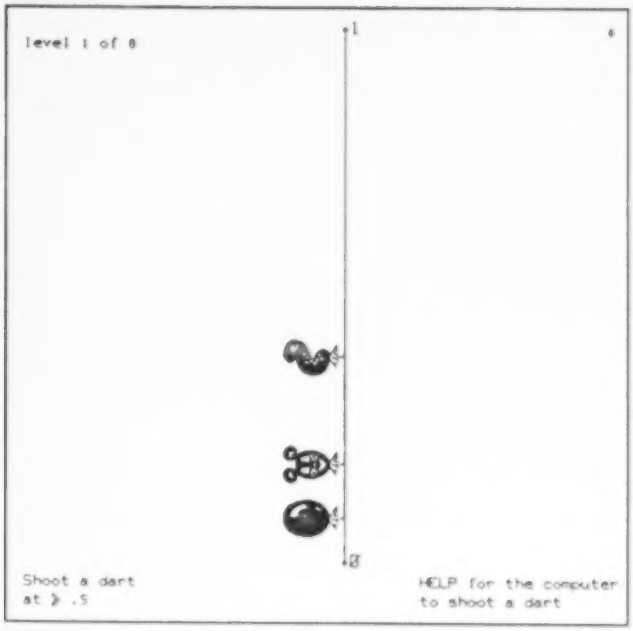



**Figure 1**

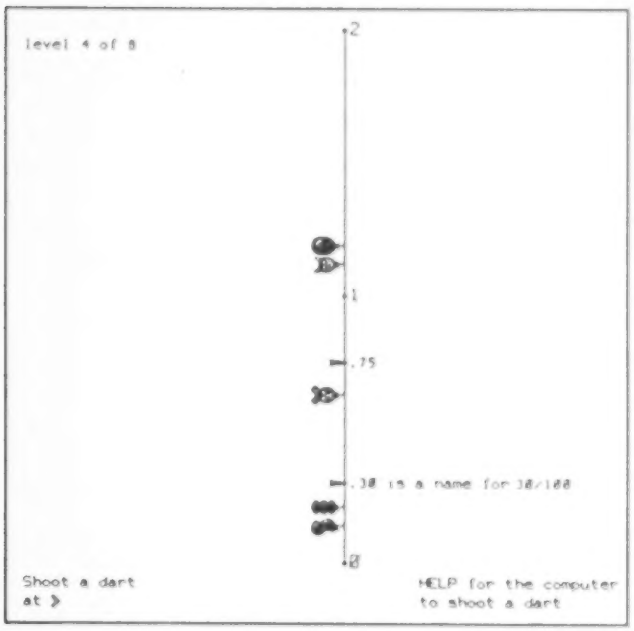



**Figure 2**

**Lesson** ## Decimal Skills I: Reading and Writing Decimals

(*SYS 4 catalog name*: **Reading and Writing Decimals**)

**File** *dec1*

**Author** Errol Magidson

**Author Affiliation** Kennedy-King College
City Colleges of Chicago
Chicago, Illinois

**Objectives**
- To read and write decimals using the place-value method.
- To read and write decimals using the point method.

**Description** Students build skills through a series of seven topics (Introduction to Decimals, Reading the Place Value Chart, Decimal Size, Place-value Method, The Computer *Reads* Your Decimal Number, The Computer *Writes* Your Decimal Value, and Point-value Method) and a series of quizzes.

**Interaction**
- Students are prompted throughout the exercises, with prompts gradually being withdrawn (see Figure 1).
- Students must continue practicing until they meet criteria.
- Students can make up problems for the computer to do (see Figure 2).

**Intended Audience** Adolescents and adults

**Completion Time** 13 minutes

**Associated Lessons**
*Decimal Skills IIa: Adding Decimals*
*Decimal Skills IIb: Subtracting Decimals*
*Decimal Skills IIIa: Multiplying Decimals*
*Decimal Skills IIIb: Dividing Decimals*
*Decimal Skills IV: Rounding Decimals*
*Decimal Skills V: Comparing Decimals and Fractions*

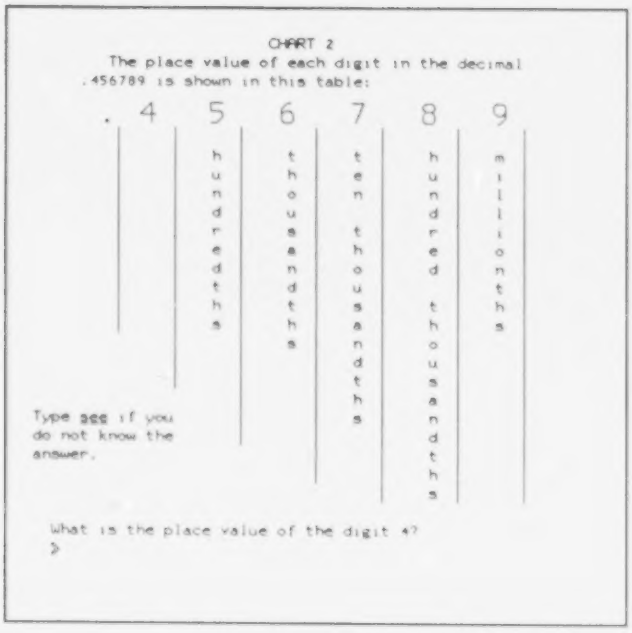


CHART 2
The place value of each digit in the decimal .456789 is shown in this table:
. 4 5 6 7 8 9
hundredths
thousandths
ten thousandths
hundred thousandths
millionths
Type see if you do not know the answer.
What is the place value of the digit 4?
>

**Figure 1**

As shown below, the computer will give the numerical equivalent of any decimal you express with its place value:
Your number
The computer's answer
Example
2 and 31 hundredths
2.31
Your number: > 5 and 3 tenths
The computer's 5.3
Press -NEXT- to type another number.
Press -END- for index.

**Figure 2**

**Lesson** **Decimal Skills IIa: Adding Decimals**

(*SYS 4 catalog name*: **Adding Decimals**)

**File** *dec2a*

**Author** Errol Magidson

**Author Affiliation** Kennedy-King College
City Colleges of Chicago
Chicago, Illinois

**Objectives**
- To line up decimals in preparation for adding.
- To find the sum of two or three decimal numbers.

**Description** Students build skills through a series of three topics (Introduction to Adding Decimals, Adding Decimals, The Computer Adds Your Decimals) and a quiz.

**Interaction**
- Students are prompted for the answers throughout the exercises (see Figure 1).
- Students must continue practicing until they meet criteria (see Figure 2).
- Students have an opportunity to make up problems for the computer to do.

**Intended Audience** Adolescents and adults

**Completion Time** 15 minutes

**Associated Lessons**
*Decimal Skills I: Reading and Writing Decimals*
*Decimal Skills IIb: Subtracting Decimals*
*Decimal Skills IIIa: Multiplying Decimals*
*Decimal Skills IIIb: Dividing Decimals*
*Decimal Skills IV: Rounding Decimals*
*Decimal Skills V: Comparing Decimals and Fractions*

Practice Exercise 1
Try placing the following numbers so they can be added properly:
775 + .657 + 5.7
The computer has typed the first number. Use the space-bar to space or -ERASE- to start over. Press -NEXT- to copy the number.
Here is the 1st number: 775.###
The second number goes here: .657 Just right!
The third number goes here: 5.7 You are off.
You are lining up your first digit in the decimal point column.
Placement Number in a row correct: # Number wrong: 1
Let's consider you an expert if you can line up 6 numbers in a row correctly.
You will receive review should you miss 3 placements

**Figure 1**

Practice Exercise 2
Try placing the following numbers so they can be added properly, and then add them:
237.4 + 36.47 + 4.732
The computer has typed the first number. Use the space-bar to space or -ERASE- to start over. Press -NEXT- to copy the number.
Here is the 1st number: 237.4##
The second number goes here: 36.47 Right in line!
The third number goes here: 4.732 Just right!
Add on computer: 267.502
Your answer is too small.
Placement Number in a row correct: 2 Number wrong: #
Addition Number in a row correct: # Number wrong: 1
Let's consider you an expert if you can find the sum of 3 problems in a row correctly.
You will receive review should you miss 3 placements or 2 additions.

**Figure 2**

**Lesson** ## Decimal Skills IIb: Subtracting Decimals

(*SYS 4 catalog name*: **Subtracting Decimals**)

**File** *dec2b*

**Author** Errol Magidson

**Author Affiliation** Kennedy-King College
City Colleges of Chicago
Chicago, Illinois

**Objectives**
- To line up decimals in preparation for subtracting.
- To find the difference, given two decimal numbers.

**Description** Students build skills through a series of three topics (Introduction to Subtracting Decimals, The Computer Subtracts Your Decimals, Special Problems) and a quiz.

**Interaction**
- Students are prompted for the answers throughout the exercises (see Figure 1).
- Students must continue practicing until they meet criteria.
- Students have an opportunity to make up problems for the computer to do (see Figure 2).

**Intended Audience** Adolescents and adults

**Completion Time** 15 minutes

**Associated Lessons**
*Decimal Skills I: Reading and Writing Decimals*
*Decimal Skills IIa: Adding Decimals*
*Decimal Skills IIIa: Multiplying Decimals*
*Decimal Skills IIIb: Dividing Decimals*
*Decimal Skills IV: Rounding Decimals*
*Decimal Skills V: Comparing Decimals and Fractions*

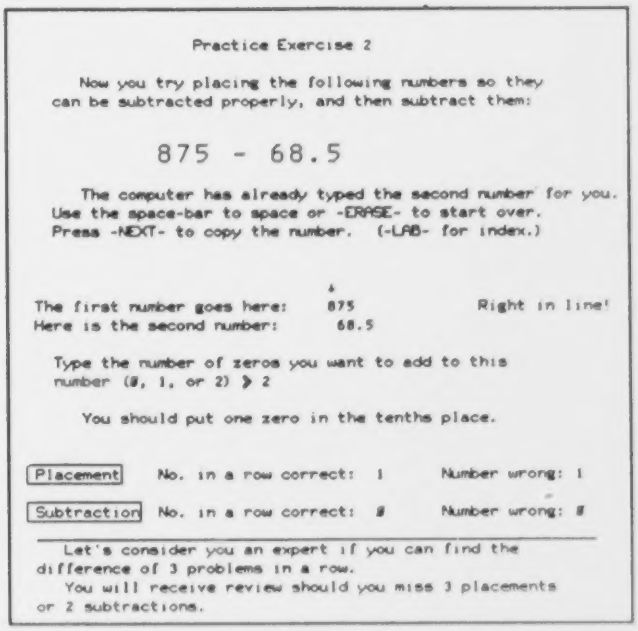

Practice Exercise 2
Now you try placing the following numbers so they can be subtracted properly, and then subtract them:
875 - 68.5
The computer has already typed the second number for you.
Use the space-bar to space or -ERASE- to start over.
Press -NEXT- to copy the number. (-LAB- for index.)
The first number goes here: 875 Right in line!
Here is the second number: 68.5
Type the number of zeros you want to add to this number (0, 1, or 2) > 2
You should put one zero in the tenths place.
Placement No. in a row correct: 1 Number wrong: 1
Subtraction No. in a row correct: 0 Number wrong: 0
Let's consider you an expert if you can find the difference of 3 problems in a row.
You will receive review should you miss 3 placements or 2 subtractions.

**Figure 1**

The computer will line up the decimals you type, and will subtract them for you. Do not use more than 5 figures in each decimal.
first decimal: 342 second decimal: > .695
Here is your first number: 342.000
Here is your second number: 0.695

**Figure 2**

**Lesson** **Decimal Pinball**

**File** *pind*

**Authors** Sharon Dugdale
David Kibbey
Helen Lueng

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To convert fractions and mixed numbers to decimals.

**Description** Students practice converting mixed numbers and fractions (with denominators 10, 100, or 1000) to decimal numbers in a game format. Students write decimal names as the ball bounces from problem to problem, as in a pinball game (see Figure 1).

**Interaction**

- Speed, as well as accuracy, is necessary for credit (see Figure 2).
- The faster students answer, the shorter the time-interval allowed for succeeding answers.
- Students earn between 20 and 50 points per problem.
- The highest scores are listed in a Hall of Fame (see Figure 3).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 15 minutes (open-ended, minimum of 10 minutes required)

**Associated Lessons** *Decimal and Fraction Conversions*
*Sort Number Names with Decimals*
*Pinball Number Facts Drill*

**Figure 1**

**Figure 2**

DECIMAL PINBALL
HALL OF FAME

| | Name | Group | Score | Date |
|---|---|---|---|---|
| 1 | mr tibbetts | tatgroup | 272# | May 9, 1989 |
| 2 | brenda | tmd | 2#9# | November 3#, 1988 |
| 3 | tibbetts | mainep | 168# | April 29, 1989 |
| 4 | michelle dessus | susd | 168# | June 19, 1989 |
| 5 | urbina julian | tmd | 154# | January 12, 199# |
| 6 | bonfiglio c | coolidge | 152# | February 24, 1989 |
| 7 | munguia alvina | tmd | 15## | January 12, 199# |
| 8 | logue mike | paradise | 149# | March 6, 1989 |
| 9 | glass p | tmd | 144# | April 27, 1989 |
| 1# | jodi knapp | cphs | 142# | January 6, 1989 |
| 11 | casey livings | caresys | 141# | June 7, 1989 |
| 12 | crystal lopez | more | 127# | July 12, 1988 |
| 13 | visitor6 | compucat | 127# | October 24, 1989 |
| 14 | carrillo richard | tmd | 126# | December 21, 1989 |
| 15 | joe higuera | tmd | 123# | January 19, 199# |
| 16 | street w | coolidge | 122# | February 24, 1989 |
| 17 | anthony, cervantes+ | redsys | 122# | May 31, 1989 |
| 18 | sargent ellen | tmd | 121# | February 22, 1989 |
| 19 | childers r | coolidge | 119# | March 3, 1989 |
| 2# | hector velasco | cpsys | 119# | May 16, 1989 |

This Hall of Fame was started on January 27, 1988.

**Figure 3**

**Lesson** ## Decimal and Fraction Conversions

**File** *con (SYS 4 file name: con.3)*

**Authors** Sharon Dugdale
David Kibbey
Helen Lueng

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objectives**
- To convert fractions and mixed numbers to decimal numbers.
- To convert decimal numbers to fractions or mixed numbers.

**Description** Students give decimal names for fractions or mixed numbers with denominators of 10, 100, or 1000; or vice versa.

**Interaction**
- Corrective feedback leads students to the correct answer (see Figure 1).
- Students must meet criteria for credit.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Decimal Pinball*
*Sort Number Names with Decimals*

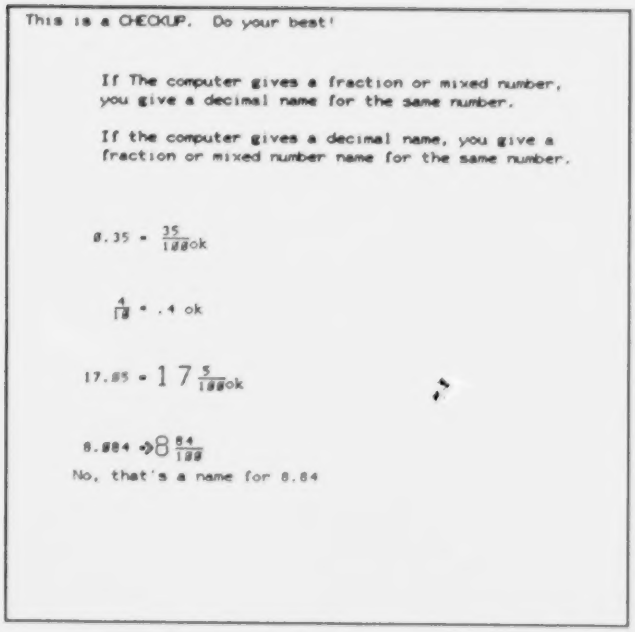

This is a CHECKUP. Do your best!
If The computer gives a fraction or mixed number, you give a decimal name for the same number.
If the computer gives a decimal name, you give a fraction or mixed number name for the same number.
0.35 = 35/100 ok
4/10 = .4 ok
17.05 = 17 5/100 ok
0.884 → 8 84/100
No, that's a name for 0.84

**Figure 1**

**Lesson** ## Sort Number Names with Decimals

**File** *sortd*

**Authors** Sharon Dugdale
David Kibbey
Helen Lueng

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To identify equivalent number names in the form of common fractions, mixed numbers, whole numbers, and decimals.

**Description** Students practice identifying equivalent fractions, mixed numbers, whole numbers, and decimals by sorting them into loops on the screen (see Figure 1).

**Interaction**

- Two to four loops appear on the screen allowing for simpler or more complicated discrimination.
- Students touch the numeral they want to move, and then, touch the loop it should be in.
- Numerals can be moved from one loop to another, if students change their minds.
- When done, students ask the computer to check their work.
- If all of the numbers are not properly placed, students are given a chance to reorganize them.

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Decimal Pinball*
*Decimal and Fraction Conversions*
*Sort Equivalent Fractions*

**Special Note** Touch capability required

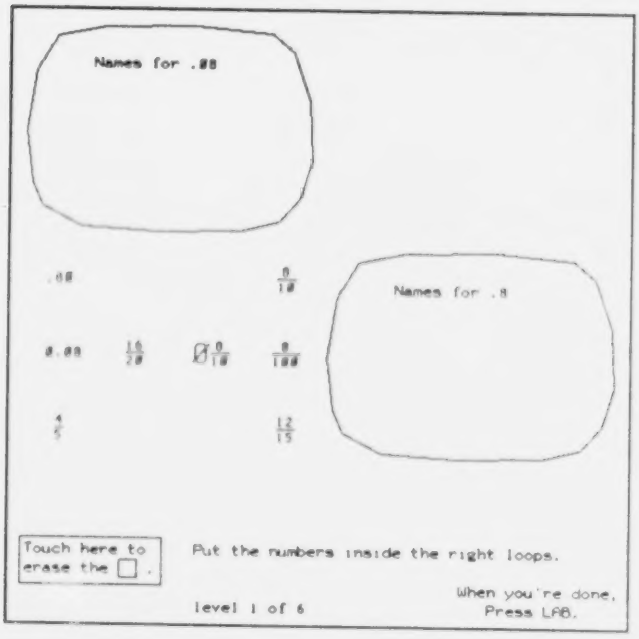

Names for .88
.88
0/10
Names for .8
8.88
10/20
Ø 0/10
8/100
4/5
12/15
Touch here to erase the □.
Put the numbers inside the right loops.
level 1 of 6
When you're done, Press LAB.

**Figure 1**

**Lesson** **Decimal Skills V: Comparing Decimals and Fractions**

(*SYS 4 catalog name*: **Comparing Decimals and Fractions**)

**File** *dec5*

**Author** Errol Magidson

**Author Affiliation** Kennedy-King College
City Colleges of Chicago
Chicago, Illinois

**Objectives**
- To change fractions and mixed numbers to decimals, and vice versa.
- To determine which of two numbers is larger.

**Description** Students build skills through a series of four topics (Changing Fractions to Decimals, Changing Mixed Numbers to Decimals, Changing Decimals to Fractions, and Comparing Decimals and Fractions) and three quizzes.

**Interaction**
- Students are prompted for the answers throughout the exercises (see Figure 1).
- Students must continue practicing until they meet criteria.
- Corrective feedback provides encouragement and leads students to the correct answer (see Figure 2).

**Intended Audience** Adolescents and adults

**Completion Time** 13 minutes

**Associated Lessons**
*Decimal Skills I: Reading and Writing Decimals*
*Decimal Skills IIa: Adding Decimals*
*Decimal Skills IIb: Subtracting Decimals*
*Decimal Skills IIIa: Multiplying Decimals*
*Decimal Skills IIIb: Dividing Decimals*
*Decimal Skills IV: Rounding Decimals*

Practice Exercise 2
Change each mixed number to a decimal. If necessary, carry out your division to the thousandths place.
39 16/46
-HELP- available for answer
> 39 .33
Your answer is too small.
Number in a row correct: 0 Number of mistakes: 1
Let's consider you an expert if you can get 3 in a row correct. You will be given review should you miss 2.

**Figure 1**

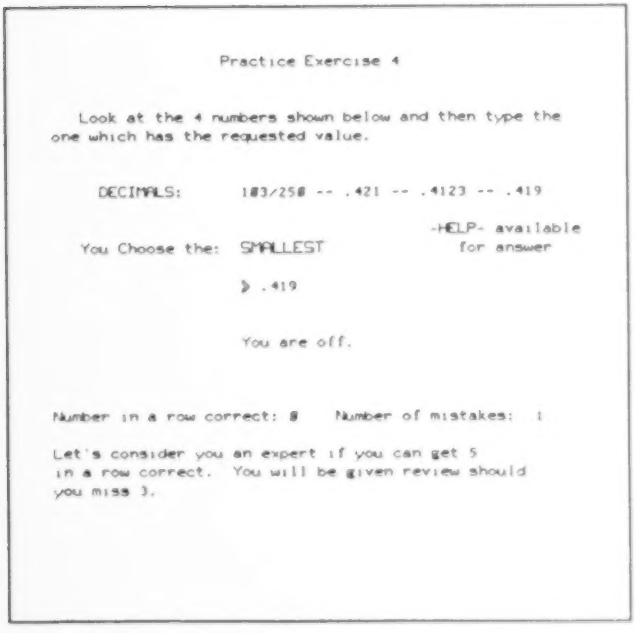

Practice Exercise 4
Look at the 4 numbers shown below and then type the one which has the requested value.
DECIMALS: 183/250 -- .421 -- .4123 -- .419
-HELP- available for answer
You Choose the: SMALLEST
> .419
You are off.
Number in a row correct: 0 Number of mistakes: 1
Let's consider you an expert if you can get 5 in a row correct. You will be given review should you miss 3.

**Figure 2**

# Unit 15

## Multiplying and Dividing Decimals

**Objective** To multiply, divide, and round off decimals.

**Description** Students are taught to multiply and divide decimals. They also learn to round off numbers.

The final lesson is a review of all four operations on both common fractions and decimal numbers.

**Lessons**

- Decimal Skills IIIa: Multiplying Decimals
  Decimal Skills IIIb: Dividing Decimals
  Decimal Skills IV: Rounding Decimals
  Math Review Drills

**Offline Materials**

- CCA Assignment Sheet
- CCA Worksheets

| | |
|---|---|
| 1A, 1B, 1C | Multiplying Decimal Numbers |
| 3A, 3B, 3C | Zero as a Place Holder - I |
| 5A, 5B, 5C | Dividing into Decimals |
| 7A, 7B, 7C | Zero as a Place Holder - II |
| 9A, 9B, 9C | Dividing by Decimal Numbers |
| 11A, 11B, 11C | Adding Zeroes to Divide |
| 13A, 13B, 13C | Rounding Off |

**Lesson** **Decimal Skills IIIa: Multiplying Decimals**

(*SYS 4 catalog name*: **Multiplying Decimals**)

**File** *dec3a*

**Author** Errol Magidson

**Author Affiliation** Kennedy-King College
City Colleges of Chicago
Chicago, Illinois

**Objective** To multiply decimals.

**Description** This four-part lesson teaches students to find the product of any two decimals and to insert the decimal point in the correct place in the product.

The first part of the lesson explains how to count the number of decimal places needed in the answer (see Figure 1). The second part teaches students to add zeroes as place holders to the product (see Figures 2 and 3). The third part of the lesson helps students practice multiplication problems and write the answers correctly (see Figure 4). The fourth part of the lesson is a quiz.

Students must not miss more than two or three answers per section in order to move to the next section. To pass the final quiz, they need to answer correctly three of three or five of six problems.

**Interaction**

- Students see a decimal problem worked by the computer.
- Students press DATA when they wish to place the decimal in the product by themselves.
- Students press HELP to see the rule for placing the decimal in the product. (This feature is not available during a quiz.)
- Students progress to working problems on their own.

**Intended Audience** Adolescents and adults

**Completion Time** 20 minutes (self-paced)

**Associated Lessons**
*Decimal Skills I: Reading and Writing Decimals*
*Decimal Skills IIa: Adding Decimals*
*Decimal Skills IIb: Subtracting Decimals*
*Decimal Skills IIIb: Dividing Decimals*
*Decimal Skills IV: Rounding Decimals*
*Decimal Skills V: Comparing Decimals and Fractions*

**Special Note** Students will need pencil and paper to complete this exercise.

You multiply decimals the same as you multiply whole numbers (32 × 15, for example), except for placing your decimal point.
The computer will show you how to decide where to place the decimal point after you multiply decimals:
Problem
Product
1 2 3
87.70 × 873.7 = 76623.490
After you have seen this example:
Press -DATA- when you think you can place the decimal point yourself.
-HELP- to see the rule.
-NEXT- for another example.

**Figure 1**

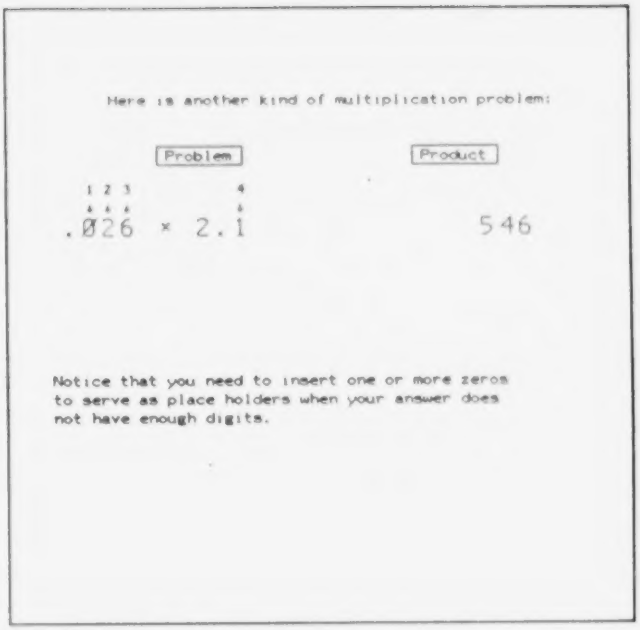

Here is another kind of multiplication problem:
Problem
Product
1 2 3 4
.026 × 2.1
546
Notice that you need to insert one or more zeros to serve as place holders when your answer does not have enough digits.

**Figure 2**

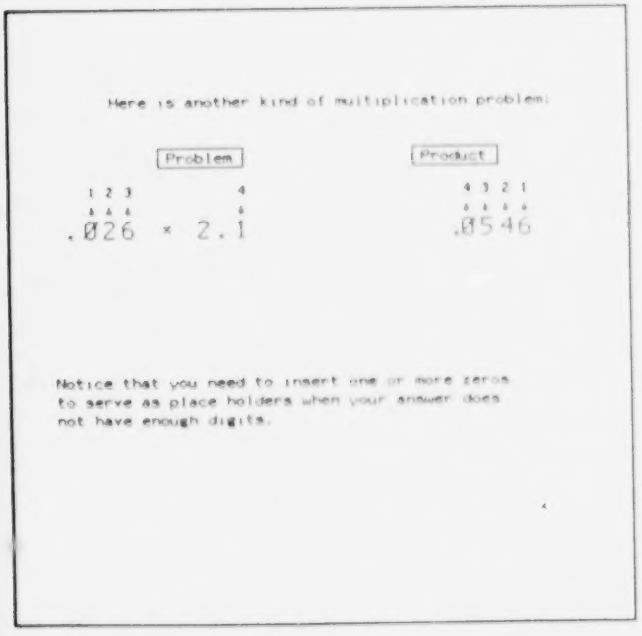

Here is another kind of multiplication problem:
Problem
Product
1 2 3 4
.026 × 2.1
4 3 2 1
.0546
Notice that you need to insert one or more zeros to serve as place holders when your answer does not have enough digits.

**Figure 3**

Practice Exercise 4
Multiply these problems. You may use paper and pencil.
Press -HELP- if you can't find the answer.
Problem
Product
.0018 × .23
Number in a row correct: 0 Number of mistakes: 0
Let's consider you an expert if you can get 3 in a row correct. You will be given help should you miss 2.

**Figure 4**

**Lesson** **Decimal Skills IIIb: Dividing Decimals**

(*SYS 4 catalog name*: **Dividing Decimals**)

**File** *dec3b*

**Author** Errol Magidson

**Author Affiliation** Kennedy-King College
City Colleges of Chicago
Chicago, Illinois

**Objective** To divide decimals.

**Description** This four-part lesson teaches students to find the quotient of two numbers where there is a decimal in the dividend, and to insert the decimal point in the correct place in the divisor (see Figure 1).

The first part of the lesson explains how to count the number of decimal places needed in the answer. The second part explains how to add zeroes as place holders when needed (see Figure 2). The third and fourth parts are quizzes over the material in the first and second parts.

To earn credit for this lesson, students must answer correctly three of three or five of six problems in both quizzes.

**Interaction**

- HELP is available when students ask for it on the practice sections. It is not available during the quiz.
- Students may press LAB to return to the lesson index at any time. They may press DATA to review any practice exercise.

**Intended Audience** Adolescents and adults

**Completion Time** 20 minutes (self-paced)

**Associated Lessons**

*Decimal Skills I: Reading and Writing Decimals*
*Decimal Skills IIa: Adding Decimals*
*Decimal Skills IIb: Subtracting Decimals*
*Decimal Skills IIIa: Multiplying Decimals*
*Decimal Skills IV: Rounding Decimals*
*Decimal Skills V: Comparing Decimals and Fractions*

**Special Note** Students will need pencil and paper to complete this exercise.

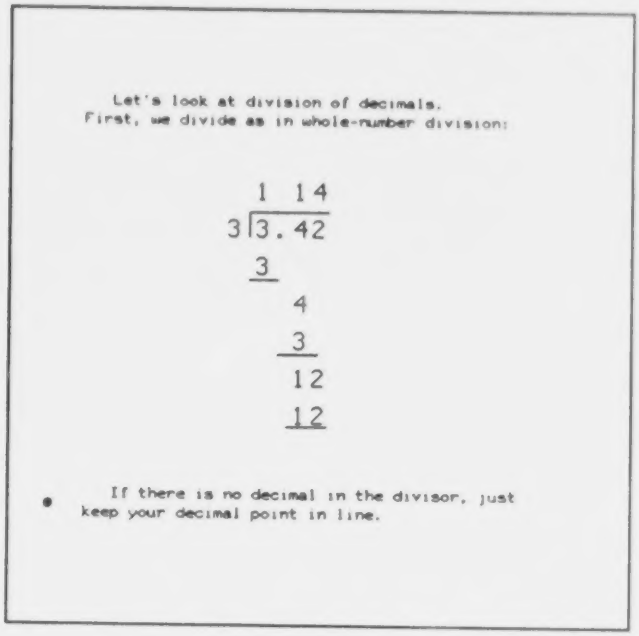



**Figure 1**

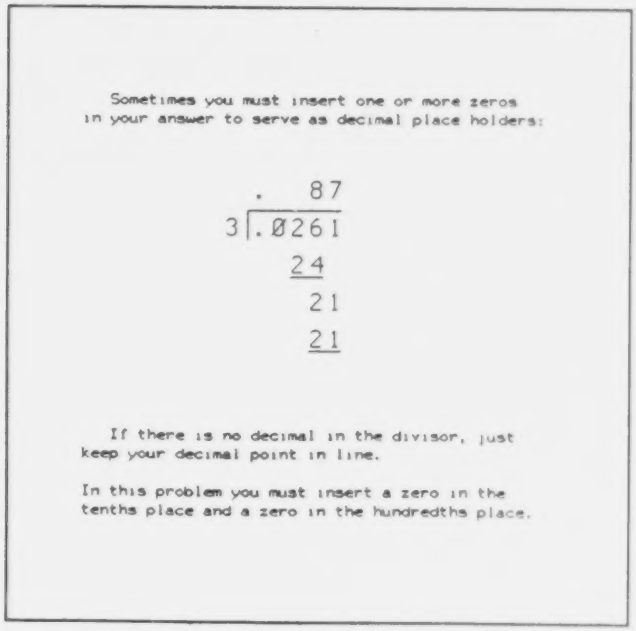

Sometimes you must insert one or more zeros in your answer to serve as decimal place holders:
. 87
3|.0261
24
21
21
If there is no decimal in the divisor, just keep your decimal point in line.
In this problem you must insert a zero in the tenths place and a zero in the hundredths place.

**Figure 2**

**Lesson** **Decimal Skills IV: Rounding Decimals**

(*SYS 4 catalog name*: **Rounding Decimals**)

**File** *dec4*

**Author** Errol Magidson

**Author Affiliation** Kennedy-King College
City Colleges of Chicago
Chicago, Illinois

**Objective** To round off decimals to the nearest tenth, hundredth, or thousandth.

**Description** This three-part lesson teaches students to round off decimals. In the first part, students are given the rules for rounding off (see Figure 1). In the second part, students practice their new skill. The last part is a quiz.

To earn credit for this lesson, students must answer correctly three of three or five of six problems in the quiz.

**Interaction**

- Students are guided through the process of rounding answers to the nearest appropriate place.
- Students may press DATA for more information on when rounding is useful in everyday situations; e.g., at the grocery store, or buying gasoline.
- Students may press NEXT for more explanatory problems in the first part, if they are not sure how rounding works.

**Intended Audience** Adolescents and adults

**Completion Time** 20 minutes (self-paced)

**Associated Lessons**
*Decimal Skills I: Reading and Writing Decimals*
*Decimal Skills IIa: Adding Decimals*
*Decimal Skills IIb: Subtracting Decimals*
*Decimal Skills IIIa: Multiplying Decimals*
*Decimal Skills IIIb: Dividing Decimals*
*Decimal Skills V: Comparing Decimals and Fractions*

Suppose you have a division problem which has no exact answer:
2504
5.31 1.330000
1330
2680
2655
2500
You could do this problem forever. Insert the decimal point and then decide how many places to carry out the division.
Although the computer has carried this problem out to the ten thousandth's place, the answer of .2504 could be correct as .250 when rounded to the nearest thousandth, or it could be correct as .25 when rounded to the nearest hundredth, or .3 when rounded to the nearest tenth.

**Figure 1**

**Lesson** **Math Review Drills**

**File** *mars*

**Authors** Noa Shinderman

**Author Affiliation** Malcolm X College
City Colleges of Chicago
Chicago, Illinois

**Objective** To add, substract, multiply, and divide common fractions and decimals.

**Description** This six-part lesson covers multiplication and division of common fractions, and addition, subtraction, multiplication, and division of decimals.

Each section begins with a brief explanation of how to work the type of problem presented (see Figure 1). Students then practice on similar problems (see Figure 2).

**Interaction**

- HELP is available to show students how to work the exercises.
- Students must complete at least four problems without help to pass each section.

**Intended Audience** Adolescents and adults

**Completion Time** 35 minutes (self-paced)

**Special Note** Students will need pencil and paper to complete this lesson.

To multiply decimals,
3.41 * 22.3
Step 1: first multiply as if they were whole numbers.
341
* 223
1#23
682
682
76#43
Step 2: Then count the total number of decimal places of the numbers being multiplied.
3.41 has 2 places
22.3 has 1 places
In total there are 3 decimal places.
Step 3: Take your answer from step 1, then from the right, count off the total number of decimal places found in step 2.
76#43
decimal places = 3
76#43
answer = 76.#43
Press -NEXT- for the drill.

**Figure 1**

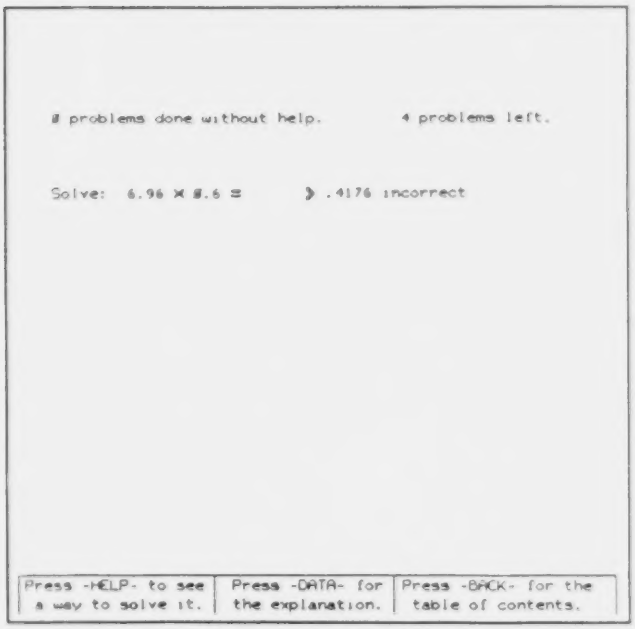

# problems done without help.
4 problems left.
Solve: 6.96 × #.6 =
) .4176 incorrect
Press -HELP- to see a way to solve it.
Press -DATA- for the explanation.
Press -BACK- for the table of contents.

**Figure 2**

# Unit 16

## Consolidation #2

**Objective** To review the skills in working with fractions taught in Units 10 through 15.

**Description** The instructors should review students' tests and assign review materials corresponding to the types of items missed. Suggested lessons for further study are listed below. All have been described in previous units except *Simplest Terms: Equivalent Fractions*, which is described in this unit.

There are no off-line worksheets specific to this unit. Instructors should make appropriate assignments from the worksheets in previous units or assign work from other appropriate materials.

NOTE: NONE OF THESE LESSONS WILL BE ASSIGNED UNLESS THE INSTRUCTOR SPECIFICALLY CHOOSES TO DO SO.

**Lessons**

- There are two groups of lessons: the first covers common fractions and mixed numbers, and the second covers decimals. Lessons assigned from the first group will appear in a sequence with review allowed (as lessons normally do in this curriculum).

  When all lessons assigned from the first group are completed, then, lessons assigned from the second group will begin to appear. At this point, the lessons from the first group will no longer be available.

  Students then work their way through the second set of lessons in the same way.

- There is one new lesson:

  *Simplest Terms: Equivalent Fractions*

- The lessons for students to review from previous units include:

| Title of Lesson | Unit Number | File Name | Page Number |
|---|---|---|---|
| **Group 1** | | | |
| {Boxes: Name Equivalent Fractions} | 10 | *recnams* (SYS 4: *recnams.1*) | 102 |
| {Add and Subtract Mixed Numbers} | 11 | *pad1* (SYS 4: *pad1.1*) | 125 |
| {Subtraction with Borrowing} | 11 | *pad2* (SYS 4: *pad2.1*) | 133 |
| {Addition by Using an Easier Problem 2} | 12 | *hs4p* (SYS 4: *hs4p.4*) | 148 |
| {High Wire} | 12 | *monkey* (SYS 4: *monkey.1*) | 152 |
| {Addition and Subtraction Drill [Unlike Denominators]} | 12 | *tryad* (SYS 4: *tryad.3*) | 154 |
| {Add and Subtract with Equivalence Sets} | 12 | *slad2* (SYS 4: *slad2.1*) | 164 |
| {Multiplication of Fractions} | 13 | *fmult* | 173 |
| {Mixed Number Multiplication Practice} | 13 | *fmdrill* | 177 |
| {Practice with Fractions} | 13 | *fracprac* | 179 |
| **Group 2** | | | |
| {Decimal Skills IIa: Adding Decimals} | 14 | *dec2a* | 186 |
| {Decimal Skills IIb: Subtracting Decimals} | 14 | *dec2b* | 188 |
| {Decimal Skills IIIa: Multiplying Decimals} | 15 | *dec3a* | 199 |
| {Decimal Skills IIIb: Dividing Decimals} | 15 | *dec3b* | 201 |
| {Decimal Skills IV: Rounding Decimals} | 15 | *dec4* | 203 |
| {Math Review Drills} | 15 | *mars* | 205 |
| **Consolidation #2** | | | 208 |


{ } Lesson will not be assigned unless the instructor chooses to do so. (See Preface for explanation.)

**Lesson** **Simplest Terms: Equivalent Fractions**

**File** *sterms (SYS 4 file name: sterms,2)*

**Authors** Sharon Dugdale
David Kibbey
Helen Leung

**Author Affiliation** CERL Mathematics Group
Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

**Objective** To practice reducing fractions to simplest terms.

**Description** This lesson leads students from the concept of equivalence sets to recognition of the simplest terms name in each set. Students then practice identifying the simplest terms name for a series of fractions.

**Interaction**

- Students identify the simplest terms name in a given set of equivalent fractions (see Figure 1).
- Students give the simplest terms name for a given fraction without the set of equivalent names available.
- Feedback for errors guides students to the correct answer (see Figure 2).

**Intended Audience** Elementary, adolescent and adult

**Completion Time** 10 minutes (self-paced)

**Associated Lessons** *Introduction to Simplest Terms*
*Checkup: Simplest Terms*

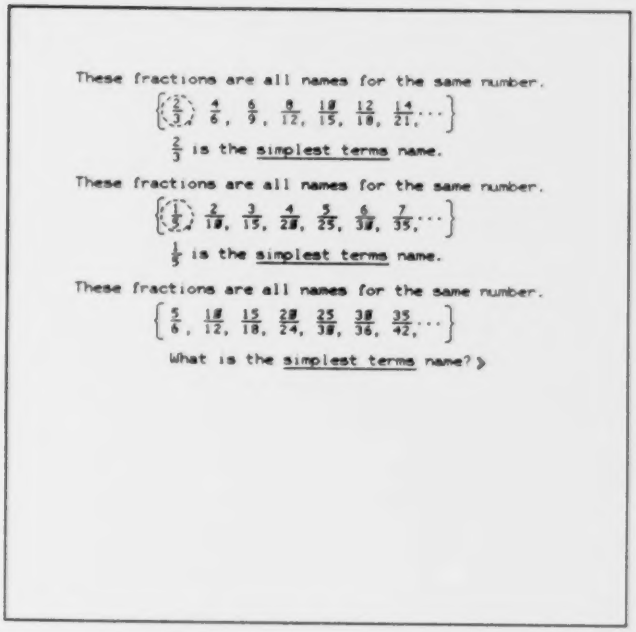

These fractions are all names for the same number.
{2/3, 4/6, 6/9, 8/12, 10/15, 12/18, 14/21, ...}
2/3 is the simplest terms name.
These fractions are all names for the same number.
{1/5, 2/10, 3/15, 4/20, 5/25, 6/30, 7/35, ...}
1/5 is the simplest terms name.
These fractions are all names for the same number.
{5/6, 10/12, 15/18, 20/24, 25/30, 30/36, 35/42, ...}
What is the simplest terms name? >

**Figure 1**

What is the simplest terms name for 20/40?
> 10/20 no
That's a name for 20/40,
but it's not in simplest terms.
HELP Available.
level 9 of 12

**Figure 2**

# APPENDIX A

## Sample Off-Line Materials

**UNIT M2 -- ASSIGNMENT SHEET**
**ADVANCED ADDITION**

**Name** _____

**Start Date** _____ **End Date** _____

**Objectives:** When you finish this unit you will be able to add numbers that require carrying. Here are some samples of problems you will learn to solve.

| | | | |
|---|---|---|---|
| | | | 9,837 |
| 36 | 766 | 856 | 4,159 |
| + 92 | + 173 | + 550 | + 3,566 |

Computer Lessons:

___ Addition Math Facts Drill*

___ Adding and Carrying

___ Addition Checkup

___ Addition & Subtraction Speedway (10 min.)

* If this lesson was completed in the previous unit, only a review is required in this unit.

© H. Barnes, 1980, 1987

Unit M2 Pencil-and-paper Assignments:

___ 1. Worksheet 1A, _____, _____ (Learning to Carry)

___ 2. Special Assignments: _____

___ 3. Worksheet 3A, _____, _____ (Carrying to the Hundreds Column)

___ 4. Special Assignments: _____

___ 5. Worksheet 5A, _____, _____ (Carrying to the Thousands and Ten Thousands Column)

___ 6. Special Assignments: _____

___ 7. Workbook Assignment: _____ (Word Problems)

You may take the Unit M2 posttest when you have completed both the computer lessons and the pencil-and-paper assignments.

© H. Barnes, 1980, 1987

UNIT M11 — ASSIGNMENT SHEET
ADDING AND SUBTRACTING FRACTIONS WITH LIKE DENOMINATORS

Name _____

Start Date _____ End Date _____

Objectives: When you finish this unit you will be able to add and subtract fractions with like denominators. Here are some samples of problems you will learn to solve.

$$\begin{array}{r} 5/12 \\ +\ 5/12 \\ \hline \end{array} \qquad 21\ \tfrac{3}{7} \qquad \begin{array}{r} 2/3 \\ -\ 1/3 \\ \hline \end{array}$$

$$-\ 18\ \tfrac{4}{7}$$

Computer Lessons:

___ Mixed Number Notation

___ New Answer Format

___ Adding and Subtracting Mixed Numbers

___ Simplifying Answers to Addition Problems

___ Standard Form Practice

___ Addition Practice: Simplifying Answers

___ Subtraction with Borrowing

___ Add and Subtract Like Denominators

___ Simplifying and Borrowing

___ Checkup: Standard Form

___ Checkup: Addition and Subtraction

© H. Barnes, 1980, 1987

Unit M11 Pencil-and-paper Assignments:

___ 1. Worksheet 1A, _____, _____ (Adding Fractions, Like Denom.)

___ 2. Special Assignments: _____

___ 3. Worksheet 3A, _____, _____ (Adding Mixed Numbers)

___ 4. Special Assignments: _____

___ 5. Worksheet 5A, _____, _____ (Subtracting Fractions, Like Denom.)

___ 6. Special Assignments: _____

___ 7. Worksheet 7A, _____, _____ (Subtracting Fractions from the Whole Number)

___ 8. Special Assignments: _____

___ 9. Worksheet 9A, _____, _____ (Subtracting Mixed Numbers)

___10. Special Assignments: _____

You may take the Unit M11 posttest when you have completed both the computer lessons and the pencil-and-paper assignments.

© H. Barnes, 1980, 1987

WORKSHEET 5C
CARRYING TO THOUSANDS AND TEN THOUSANDS COLUMN

Name _____ Date _____

Directions: Solve these problems and give to T.A. for scoring.

REVIEW

a) 743 + 389

b) 655 + 472

c) 6476 + 2718

d) 5,384 + 1,832

e) 4,546 + 3,823

f) 2,001 + 2,999

g) 6,347 + 5,736

h) 7614 + 5786

i) 7539 + 2864

j) 6,302 + 5,818

k) 8,576 + 2,533

l) 6473 + 2755

m) 38993 + 22108

n) 54935 + 29369

o) 29646 + 31374

p) 5786 + 2317

q) 1397 + 8453 + 9912

r) 7707 + 8007 + 7698

s) 6718 + 9684 + 1986

t) 6247 + 7156 + 7635

SCORE: _____

18-20: Go to Step 7

0-17: Go to TEACHER

© H. Barnes, 1980, 1987

WORKSHEET 7A
SUBTRACTING FRACTIONS FROM THE WHOLE NUMBER

Name _____ Date _____

Directions: Solve these problems and give to [T.A.] for scoring.

REVIEW

a) $\begin{array}{r} 5/9 \\ -\ 4/9 \\ \hline \end{array}$ b) $\begin{array}{r} 7/20 \\ -\ 3/20 \\ \hline \end{array}$ c) $\begin{array}{r} 1 \\ -\ 2/7 \\ \hline \end{array}$ d) $\begin{array}{r} 1 \\ -\ 5/9 \\ \hline \end{array}$ e) $\begin{array}{r} 2 \\ -\ 6/13 \\ \hline \end{array}$

f) $\begin{array}{r} 1 \\ -\ 7/9 \\ \hline \end{array}$ g) $\begin{array}{r} 6 \\ -\ 2/3 \\ \hline \end{array}$ h) $\begin{array}{r} 15 \\ -\ 3/4 \\ \hline \end{array}$ i) $\begin{array}{r} 7 \\ -\ 3/10 \\ \hline \end{array}$ j) $\begin{array}{r} 9 \\ -\ 2/5 \\ \hline \end{array}$

k) 13 - 1/8=

l) 15 - 3/8=

m) 12 - 9/9=

n) $8 - \frac{7}{11} =$

o) $69 - \frac{7}{10} =$

p) $63 - \frac{7}{9} =$

q) $\begin{array}{r} 20 \\ -\ 4/5 \\ \hline \end{array}$ r) $\begin{array}{r} 16 \\ -\ 2/3 \\ \hline \end{array}$ s) $\begin{array}{r} 23 \\ -\ 9/11 \\ \hline \end{array}$

t) Alice lives 10 miles from town. Betty lives 7/9 of a mile closer to town. How far from town does Betty live? _____

SCORE: _____

18-20: Go to Step 9

14-17: Go to Step 8

0-13: Go to TEACHER

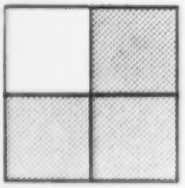
© H. Barnes, 1980, 1987

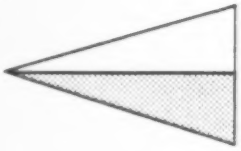
WORKSHEET 1B
WHAT IS A FRACTION?

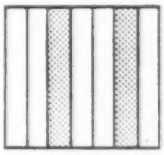
Name _____ Date _____

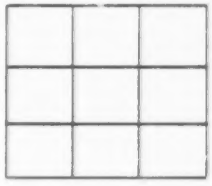
Directions: Read the instructions carefully, work the problems, and give these sheets to the [TEACHER] for scoring.

**Tell what fraction of each figure is shaded:**

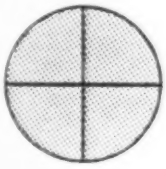
a)  ___

b)  ___

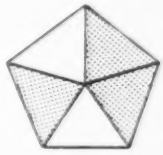
c)  ___

d)  ___

e) ___

f)  ___

g) ___

h)  ___

**Shade in the given fraction for each figure:**

i) $\frac{1}{10}$

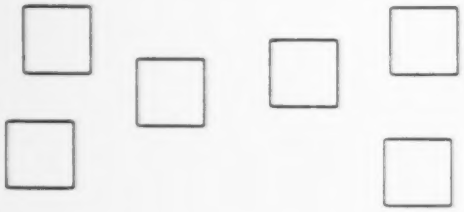
j) $\frac{2}{3}$

k) $\frac{2}{2}$

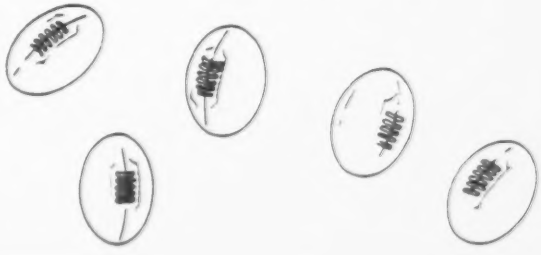
SHOW YOUR WORKING LINES

l) $\frac{5}{8}$

m) Shade in 1/6 of the squares.



n) Circle 3/5 of the footballs.

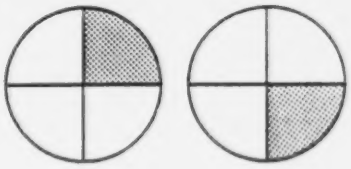


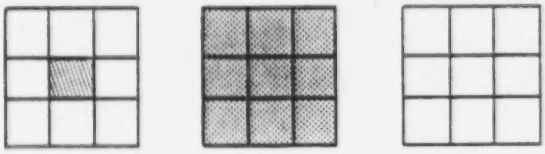
© H. Barnes, 1980, 1987

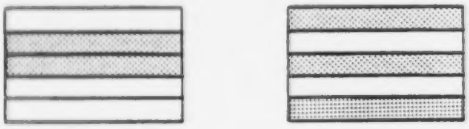


(over)

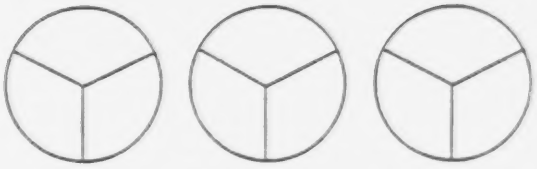
Here's a picture: What fraction is shaded?

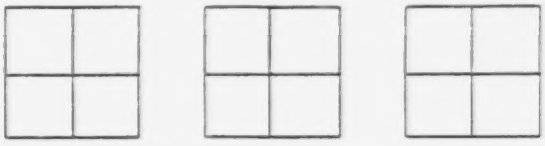
o)  =

p)  =

q)  =

Here's a fraction: Shade the picture:

r) $\frac{1}{3}$ = 

s) $\frac{3}{4}$ = 

t) $\frac{12}{8}$ =

SCORE: _____

18-20: Go to Step 3

0-17: Go to TEACHER

© H. Barnes, 1980, 1987

APPENDIX B

**Alphabetical List of Files**

**Used in**

**The CCA Basic Skills Mathematics Curriculum**


| File Name | Page Number |
|---|---|
| addchk | 10 |
| antwar | 43 |
| arrays | 30 |
| arrays2 | 41 |
| carry | 8 |
| con | 192 |
| ddarts | 182 |
| dec1 | 184 |
| dec2a | 186 |
| dec2b | 188 |
| dec3a | 199 |
| dec3b | 201 |
| dec4 | 203 |
| dec5 | 196 |
| div1 | 59 |
| div2 | 64 |
| divalg | 66 |
| divmeans | 53 |
| easyspeed | 12, 18 |
| egen | 160 |
| factspcp | 4, 7, 16, 22, 26, 40, 51, 63 |
| fmdrill | 177 |
| fmint | 169 |
| fmpix | 171 |
| fmstrat | 175 |
| fmult | 173 |
| fnotation | 76 |
| fracprac | 179 |
| gimme | 55 |
| hsrev | 146 |
| hs4p | 148 |
| lengths | 2 |
| lites | 78, 84 |
| liteseq | 80, 100 |

| File Name | Page Number |
|---|---|
| mars | 205 |
| monkey | 152 |
| multa1 | 32 |
| multa2 | 45 |
| multalg | 47 |
| multmeans | 34 |
| naf | 123 |
| nlintrob | 121 |
| nlintroc | 92 |
| pad1 | 125, 127, 129, 131, 139 |
| pad2 | 133, 137, 141 |
| pcd | 112 |
| pind | 190 |
| pinw | 70 |
| pizza4 | 86 |
| pmix | 96 |
| rec | 74 |
| recadnolib | 144, 150, 156 |
| recchk | 82 |
| recnams | 102 |
| sea | 23 |
| slad | 162 |
| slad2 | 164 |
| sortd | 194 |
| sorteqf | 104, 114 |
| sorteqm | 106 |
| sortf | 94 |
| speedway | 36, 49, 61, 68 |
| stamp | 28 |
| sterms | 118, 209 |
| sti | 110 |
| sticksv | 20 |
| strad | 158, 166 |
| tryad | 135, 154 |
| tryeq | 88, 90, 98, 108, 116 |
| trynum | 57 |

END

U.S. Dept. of Education

Office of Education
Research and
Improvement (OERI)

ERIC

Date Filmed

March 29, 1991